4.2

July 16th

① $y = (3x^2+2)(2x-1)$

$y' = (3x^2+2)'(2x-1) + (3x^2+2)(2x-1)'$

$= (6x)(2x-1) + (3x^2+2)(2) = 18x^2 - 16x + 4$

⑦ $y = (\sqrt{x}+2)(x+1)$

$y' = (\sqrt{x}+2)'(x+1) + (\sqrt{x}+2)(x+1)'$

$= \left(\frac{1}{2}x^{-1/2}\right)(x+1) + (\sqrt{x}+2)(1)$

$= \frac{3}{2}\sqrt{x} + 2 + \frac{1}{2\sqrt{x}}$

⑨ $p(y) = (y^{-1}+y^{-2})(2y^{-3}-5y^{-4})$

$p(y) = (y^{-1}+y^{-2})'(2y^{-3}-5y^{-4}) + (y^{-1}+y^{-2})(2y^{-3}-5y^{-4})'$

$= (-y^{-2}-2y^{-3})(2y^{-3}-5y^{-4}) + (y^{-1}+y^{-2})(-6y^{-4}+20y^{-5})$

$= -2y^{-5} + 5y^{-6} - 4y^{-6} + 10y^{-7}$

$-6y^{-5} + 20y^{-6} - 6y^{-6} + 20y^{-7}$

$= -8y^{-5} + 15y^{-6} + 30y^{-7}$

(14) (oops) $y = \dfrac{9-7t}{1-t}$ $y' = \dfrac{(1-t)(9-7t)' - (1-t)'(9-7t)}{(1-t)^2}$

$= \left[(1-t)(-7) - (-1)(9-7t)\right] / (1-t)^2$

$= (-7+7t+9-7t)/(1-t)^2$

$= \dfrac{2}{(1-t)^2}$

(15) $y = \dfrac{x^2+x}{x-1}$

$y' = \left[(x-1)(x^2+x)' - (x^2+x)(x-1)'\right] / (x-1)^2$

$= \left[(x-1)(2x+1) - (x^2+x)(1)\right] / (x-1)^2$

$= (2x^2+x-2x-1-x^2-x)/(x-1)^2 = (x^2-2x-1)/(x-1)^2$

(17) $f(t) = \dfrac{4t^2+11}{t^2+3}$ $f'(t) = \dfrac{(t^2+3)(8t) - (4t^2+11)(2t)}{(t^2+3)^2}$

$= \dfrac{8t^3+24t-8t^3-22t}{(t^2+3)^2}$

$= \dfrac{2t}{(t^2+3)^2}$

(21) $p(t) = \dfrac{\sqrt{t}}{t-1}$, $p'(t) = \left[(t-1)\frac{1}{2}t^{-1/2} - t^{1/2}(1)\right] / (t-1)^2$

$= \left(\dfrac{\sqrt{t}}{2} - \dfrac{1}{2\sqrt{t}} - \sqrt{t}\right) / (t-1)^2$

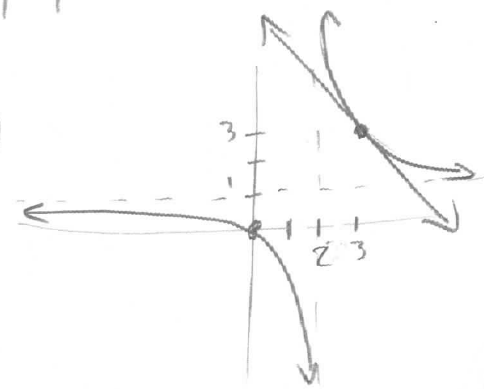
$= -\dfrac{\sqrt{t} + \frac{1}{\sqrt{t}}}{2(t-1)^2}$

(29) $g(3)=4,\ g'(3)=5,\ f(3)=9,\ f'(3)=8,\ h(x)=f(x)\,g(x)$

$$h'(3)=f'(3)\,g(3)+f(3)\,g'(3)$$

$$=8\cdot4+9\cdot5=77$$

(33) tangent line to the graph of $f(x)=\dfrac{x}{x-2}$ through $(3,3)$



$$f'(x)=\frac{(x-2)(1)-x(1)}{(x-2)^2}=\frac{-2}{(x-2)^2}$$

$$f'(3)=\frac{-2}{(3-2)^2}=-2$$

$$y-3=-2(x-3)$$

$$\boxed{y=-2x+9}$$

(38) If $f(x)=\dfrac{u(x)}{v(x)}$, then $u(x)=f(x)\,v(x)$.

$u'(x)=f'(x)\,v(x)+f(x)\,v'(x)$ by the product rule.

Solving for $f'(x)$, we get $f'(x)=\dfrac{u'(x)-f(x)\,v'(x)}{v(x)}$

$$=\frac{u'(x)-\frac{u(x)}{v(x)}v'(x)}{v(x)}=\frac{u'(x)\,v(x)-u(x)\,v'(x)}{v(x)}$$

So the product rule implies the quotient rule.

(41) $C(x) = \dfrac{3x+2}{x+4}$ $\quad \overline{C}(x) = \dfrac{3x+2}{x^2+4x}$ (c)

(a) $\overline{C}(10) = \dfrac{32}{140} \sim .2286, \$22.86$

(b) $\overline{C}(20) = \dfrac{62}{480} \sim .1292, \$12.92$

(d) $\overline{C}'(x) = \left[(x^2+4x)(3)-(3x+2)(2x+4)\right] \big/ (x^4+4x)^2$

$= (-3x^2-4x-8) \big/ (x^4+4x)^2$

(43) $M(d) = \dfrac{100d^2}{3d^2+10}$

(a) $M'(d) = \dfrac{(3d^2+10)(200d) - 100d^2(6d)}{(3d^2+10)^2}$

$= \dfrac{1200d^3+2000d}{(3d^2+10)^2}$

(b) $M'(2) = 28.1$ bicycles/day per day (after 2 days, you make 28.1 more bicycles per day)

$M'(5) = 22.1$ bicycles/day per day (after 5 days you make 22.1 more bicycles per day)

(the rate decreases, i.e. you don't keep improving production as fast)