3.4 July 14th

(11) $f(x)=3x-7$, $f'(x)=\lim_{h\to 0}\frac{(3(x+h)-7)-(3x-7)}{h}=\lim_{h\to 0}\frac{3h}{h}=3$

$f'(-2)=f'(0)=f'(3)=3$

(13) $f(x)=-4x^2+9x+2$

$f'(x)=\lim_{h\to 0}\frac{[-4(x+h)^2+9(x+h)+2]-[-4x^2+9x+2]}{h}$

$=\lim_{h\to 0}\frac{-8xh-4h^2+9h}{h}=\lim_{h\to 0}-8x+9-4h=-8x+9$

$f'(-2)=25$, $f'(0)=9$, $f'(3)=-15$

(15) $f(x)=\frac{12}{x}$, $f'(x)=\lim_{h\to 0}\left(\frac{12}{x+h}-\frac{12}{x}\right)\frac{1}{h}$

$=\lim_{h\to 0}\frac{12x-12(x+h)}{x(x+h)h}=\lim_{h\to 0}\frac{-12}{x(x+h)}=\frac{-12}{x^2}$

$f'(-2)=-3$, $f'(0)$ undefined, $f'(3)=-\frac{4}{3}$

(17) $f(x)=\sqrt{x}$, $f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

$=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\frac{(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$

$=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$ $f'(-2)$ undefined $f'(3)=\frac{1}{2\sqrt{3}}$

$f'(0)$ undefined

(19) $f(x) = 2x^3+5$, $f'(x) = \lim_{h \to 0} \frac{2(x+h)^3+5-(2x^3+5)}{h}$

$= \lim_{h \to 0} \left[2(x^3+3x^2h+3xh^2+h^3)+5-2x^3-5\right]\frac{1}{h}$

$= \lim_{h \to 0} (6x^2h+6xh^2+2h^3)\frac{1}{h}$

$= \lim_{h \to 0} 6x^2+6xh+2h^2 = 6x^2$

$f'(-2) = 24$, $f'(0) = 0$, $f'(3) = 54$

(21) $f(x) = x^2+2x$. • secant line through $(3, f(3)), (5, f(5))$

has slope $\frac{f(5)-f(3)}{5-3} = 10$, eqn $y-15 = 10(x-3)$

$\underline{y = 10x-15}$

• tangent line through $(3, f(3))$

has slope $\lim_{h \to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h \to 0} \frac{(3+h)^2+2(3+h)-15}{h}$

$= \lim_{h \to 0} \frac{6h+h^2+2h}{h} = \lim_{h \to 0} 8+h = 8 = f'(3)$

eqn $y-15 = 8(x-3)$

$\underline{y = 8x-9}$

(23) $f(x) = \frac{5}{x}$ slope of secant line through $(2, f(2))$, $(5, f(5))$ is $\dfrac{\frac{5}{5} - \frac{5}{2}}{5-2} = -\frac{1}{2}$, eqn $y - \frac{5}{2} = -\frac{1}{2}(x-2)$

$\underline{y = -\frac{x}{2} + \frac{7}{2}}$

Slope of tangent line through $(2, f(2))$ is

$$\lim_{h \to 0} \frac{\frac{5}{2+h} - \frac{5}{2}}{h} = \lim_{h \to 0} \frac{10 - 10 - 5h}{2(2+h)h} = \lim_{h \to 0} \frac{-5}{2(2+h)}$$

$= \frac{-5}{4}$ eqn $y - \frac{5}{2} = \frac{-5}{4}(x-2)$

$\underline{y = -\frac{5}{4}x + 5}$

(25) $f(x) = 4\sqrt{x}$ slope of secant line through $(9, f(9)), (16, f(16))$ is $\dfrac{4\sqrt{16} - 4\sqrt{9}}{16-9} = \dfrac{16-12}{7} = \dfrac{4}{7}$

eqn $y - 12 = \frac{4}{7}(x-9)$

$\underline{y = \frac{4}{7}x + \frac{48}{7}}$

Slope of tangent line through $(9, f(9))$ is

$$\lim_{h \to 0} \frac{4\sqrt{9+h} - 4\sqrt{9}}{h} = \lim_{h \to 0} \frac{4}{h} \frac{\sqrt{9+h} - 3}{1} \frac{\sqrt{9+h} + 3}{\sqrt{9+h} + 3}$$

$= \lim_{h \to 0} \frac{4}{\sqrt{9+h} + 3} = \frac{2}{3}$ eqn $y - 12 = \frac{2}{3}(x-9)$

$\underline{y = \frac{2}{3}x + 6}$

(37) $x = -3$ (vertical tangent line, "$f'(x) = -\infty$")

$x = -1$ (discontinuous)

$x = 0$ ($f(0)$ undefined)

$x = 2$ ($f(2)$ undefined)

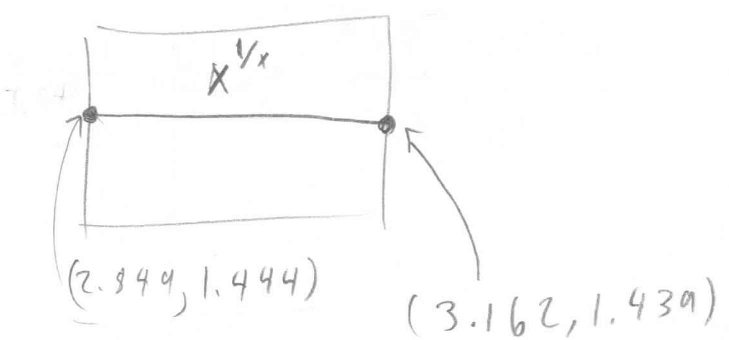
$x = 3, 5$ (f has a "corner")

(41) (a) is the distance, (b) is the velocity

(e.g. horizontal tangents ($f'(x) = 0$) in (a) correspond to zeros in (b))

(45) $f(x) = x^{1/x}$ $\quad f'(3) \approx$

x	$\dfrac{x^{1/x} - 3^{1/3}}{x - 3}$
3.01	−0.0160
3.001	−0.0158
3.0001	−0.0158



$$\frac{1.439 - 1.444}{3.162 - 2.844} \approx -0.0160$$