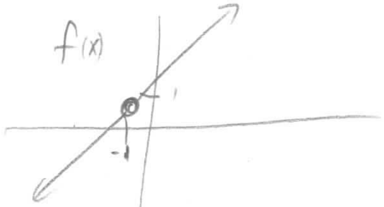
3.2 July 11th

① 

discontinuous at $x=-1$

- $f(-1)$ d.n.e.
- $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x) = \frac{1}{2} = \lim_{x\to -1} f(x)$

- $\lim_{x\to -1} f(x)$ exists, but $\neq f(-1)$ (which is undefined)

③

⑦ $f(x) = \frac{5+x}{x(x-2)}$ is discontinuous at $x=0, 2$ (vertical asymptotes)

$\lim_{x\to 2^+} f(x) = +\infty$ $\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 0^+} f(x) = -\infty$ $\lim_{x\to 0^-} f(x) = +\infty$

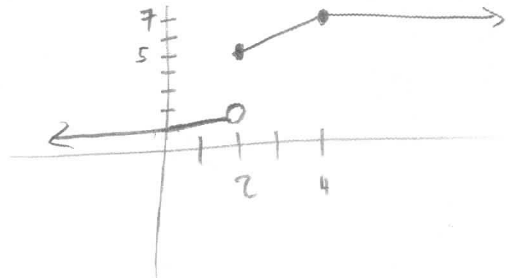
⑨ $f(x) = \frac{x^2-4}{x-2}$ is discontinuous at $x=2$ ("hole" in graph)

$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2} x+2 = 4$

⑬ $p(x) = \frac{|x+2|}{x+2} = \begin{cases} 1 & x > -2 \\ -1 & x < -2 \\ \text{undef.} & x=-2 \end{cases}$ discontinuous at $x=-2$

$\lim_{x\to -2^-} p(x) = -1$, $\lim_{x\to -2^+} p(x) = 1$, $\lim_{x\to -2} p(x)$ d.n.e.

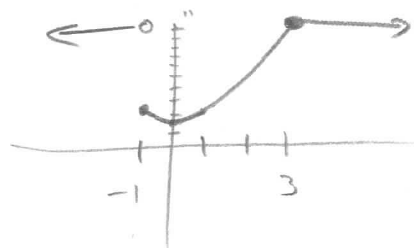
(19) $f(x) = \begin{cases} 1 & x < 2 \\ x+3 & 2 \le x \le 4 \\ 7 & x > 4 \end{cases}$



discontinuous at $x = 2$

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} 1 = 1$, $\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} x+3 = 5$

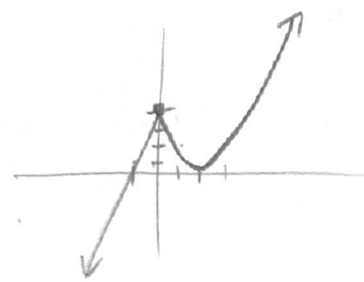
(21) $g(x) = \begin{cases} 11 & x < -1 \\ x^2+2 & -1 \le x \le 3 \\ 11 & x > 3 \end{cases}$



discontinuous at $x = -1$

$\lim\limits_{x \to -1^-} g(x) = \lim\limits_{x \to -1^-} 11 = 11$, $\lim\limits_{x \to -1^+} g(x) = \lim\limits_{x \to -1^+} x^2+2 = 3$

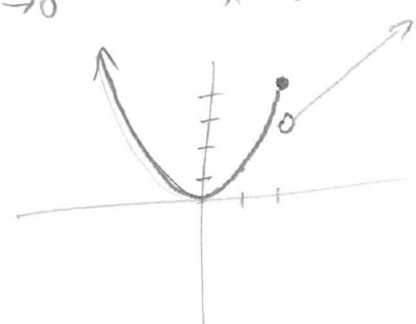
(23) $h(x) = \begin{cases} 4x+4 & x \le 0 \\ x^2-4x+4 & x > 0 \end{cases}$



continuous everywhere

$4 = h(0) = \lim\limits_{x \to 0^-} h(x) = \lim\limits_{x \to 0^-} 4x+4 = \lim\limits_{x \to 0^+} h(x) = \lim\limits_{x \to 0^+} x^2-4x+4$

(25) $f(x) = \begin{cases} kx^2 & x \le 2 \\ x+k & x > 2 \end{cases}$



To be continuous at $x = 2$,

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} kx^2 = 4k \stackrel{?}{=} \lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} x+k = 2+k$

$4k = 2+k \Rightarrow k = 2/3$

3.3 July 11th

① $y = x^2+2x$, $[1,3]$ $\quad \dfrac{(3^2+2\cdot 3)-(1^2+2\cdot 1)}{3-1} = 6$

③ $y = -3x^3+2x^2-4x+1$ $[-2,1]$

$$\frac{1}{1-(-2)}\left(\left[-3(1)^3+2(1)^2-4(1)+1\right]-\left[-3(-2)^3+2(-2)^2-4(-2)+1\right]\right)$$

$= -15$

⑤ $y = \sqrt{x}$, $[1,4]$ $\quad \dfrac{\sqrt{4}-\sqrt{1}}{4-1} = \dfrac{1}{3}$

⑮ $f(x) = x^2+2x$ at $x=0$

$$\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{h^2+2h-0}{h} = \lim_{h\to 0} h+2 = 2$$

⑰ $g(t) = 1-t^2$ at $t=-1$

$$\lim_{t\to -1} \frac{g(t)-g(-1)}{t-(-1)} = \lim_{t\to -1} \frac{1-t^2-(1-(-1)^2)}{t+1} = \lim_{t\to -1} \frac{1-t^2}{t+1}$$

$$= \lim_{t\to -1} \frac{(1-t)\cancel{(1+t)}}{\cancel{t+1}} = \lim_{t\to -1} 1-t = 2$$

⑲ $f(x) = x^x$ at $x=2$ $\quad \displaystyle\lim_{x\to 2} \frac{x^x - 2^2}{x-2} = ?$

$\frac{x^x-4}{x-2}$	6.840	6.779	6.773	6.773
x	2.01	2.001	2.0001	2.00001

≈ 6.773