3.1 July 10th

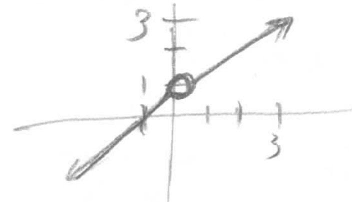
① $\lim_{x\to 2^-} f(x)=5$, $\lim_{x\to 2^+} f(x)=6$

$\lim_{x\to 2} f(x)$ dne ⓒ

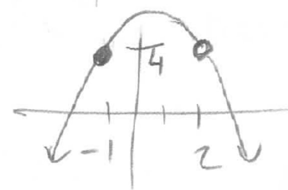
③ $\lim_{x\to 4^-} f(x)=\lim_{x\to 4^+} f(x)=6$, $f(4)$ undefined

$\lim_{x\to 4} f(x)=6$ ⓑ

⑤ $\lim_{x\to 3} f(x)=3$, $\lim_{x\to 0} f(x)=1$



⑦ $\lim_{x\to 2} F(x)=4$, $\lim_{x\to -1} F(x)=4$



$\lim_{x\to 4} f(x)=9$, $\lim_{x\to 4} g(x)=27$

(21) $\lim_{x\to 4} f(x)-g(x) = 9-27 = -18$

$= \lim_{x\to 4} f(x) - \lim_{x\to 4} g(x)$

(23) $\lim_{x\to 4} \frac{f(x)}{g(x)} = \frac{\lim_{x\to 4} f(x)}{\lim_{x\to 4} g(x)} = \frac{9}{27} = \frac{1}{3}$

(25) $\lim_{x\to 4} \sqrt{f(x)} = \sqrt{\lim_{x\to 4} f(x)} = \sqrt{9} = 3$

(27) $\lim_{x\to 4} 2^{f(x)} = 2^{\lim_{x\to 4} f(x)} = 2^9 = 512$

(29) $\lim_{x\to 4} \frac{f(x)+g(x)}{2g(x)} = \frac{\lim_{x\to 4} f(x) + \lim_{x\to 4} g(x)}{2\lim_{x\to 4} g(x)} = \frac{9+27}{2\cdot 27} = \frac{2}{3}$

(31) $\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+3)}{x-3}$

$= \lim_{x\to 3} x+3 = 6$

(35) $\lim_{x\to -2} \frac{x^2-x-6}{x+2} = \lim_{x\to -2} \frac{(x-3)(x+2)}{x+2}$

$= \lim_{x\to -2} x-3 = -5$

(37) $\lim_{x\to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x} = \lim_{x\to 0} \frac{3-(x+3)}{3(x+3)x}$

$= \lim_{x\to 0} \frac{-1}{3(x+3)} = \frac{-1}{3(0+3)} = \frac{-1}{9}$

(41) $\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h}$

$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} 2x+h = 2x$

(43) $\lim_{x\to\infty} \frac{3x}{7x-1} = \lim_{x\to\infty} \frac{3}{7-\frac{1}{x}} = \frac{3}{7-\underbrace{\lim_{x\to\infty}\frac{1}{x}}_{=0}}$

$= \frac{3}{7}$

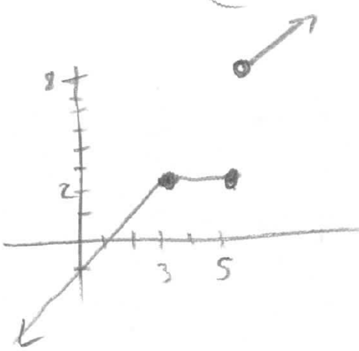
(47) $$\lim_{x\to\infty} \frac{3x^3+2x-1}{2x^4-3x^3-2} = \lim_{x\to 0} \frac{\frac{3}{x}+\frac{2}{x^3}-\frac{1}{x^4}}{2-\frac{3}{x}-\frac{2}{x^4}}$$

$$= \frac{0+0-0}{2-0-0} = 0 \quad \left(\lim_{x\to\infty} \frac{1}{x^n} = 0 \text{ for } n>0\right)$$

(49) $$\lim_{x\to\infty} \frac{2x^3-x-3}{6x^2-x-1} = \lim_{x\to\infty} \frac{2x-\frac{1}{x}-\frac{3}{x^2}}{6-\frac{1}{x}-\frac{1}{x^2}}$$

$$= \frac{\lim_{x\to\infty} 2x - 0 - 0}{6-0-0} = \infty \text{ (D.N.E.)}$$

(55) $$f(x) = \begin{cases} x-1 & x<3 \\ 2 & 3 \le x \le 5 \\ x+3 & x>5 \end{cases}$$



(a) $\lim_{x\to 3} f(x) = 2$

since $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x-1 = 3-1=2$

and $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2 = 2$

(b) $\lim_{x\to 5} f(x)$ d.n.e since $\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} 2 = 2$

and $\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} x+3 = 5+3 = 8 \neq 2$

(83) (a) $3¢ = \lim_{x\to 53} T(x)$

(b) $\lim_{x\to 09^-} T(x) = 7.25¢$

(c) $\lim_{x\to 09^+} T(x) = 8.25¢$

(d) $\lim_{x\to 09} T(x)$ d.n.e $(8.25 \neq 7.25)$

(e) $T(09) = 8.25¢$