Ch5

(19) $f(x) = -x^3 + 2x^2 + 15x + 16$

$f'(x) = -3x^2 + 4x + 15 = 0, \quad x = \dfrac{-4 \pm \sqrt{16+180}}{-6}$

$= \dfrac{2}{3} \pm \dfrac{7}{3} = 3, -5/3$

f': $-$ $+$ $-$

$-5/3$ 3

increasing

$(-5/3, 3)$

decreasing

$(-\infty, -5/3), (3, \infty)$

(24) $f(x) = 8xe^{-4x}$

$f'(x) = 8x(-4e^{-4x}) + e^{-4x}(8)$

$= 8e^{-4x}(1-4x) = 0$ at $x = \frac{1}{4}$

f': $+$ $-$

$1/4$

increasing $(-\infty, \frac{1}{4})$

decreasing $(\frac{1}{4}, \infty)$

(31) $f(x) = \dfrac{xe^x}{x-1}$ $f'(x) = \dfrac{(x-1)(xe^x + e^x) - xe^x}{(x-1)^2}$

$= \dfrac{e^x(x^2 - x - 1)}{(x-1)^2} = 0$ at $x = \dfrac{1 \pm \sqrt{1+4}}{2}$

$= \dfrac{1 \pm \sqrt{5}}{2}$

(undefined at $x = 1$ as is f)

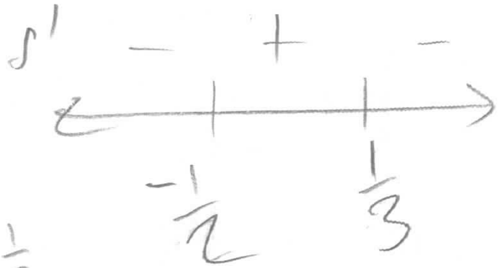
f': $+$ $-$ $-$ $+$

$\frac{1-\sqrt{5}}{2}$ 1 $\frac{1+\sqrt{5}}{2}$

- max at $\frac{1-\sqrt{5}}{2}$ of .206
- min at $1+\sqrt{5}/2$ of 13.203

(39) $f(x) = -2x^3 - \frac{x^2}{2} + x - 3$

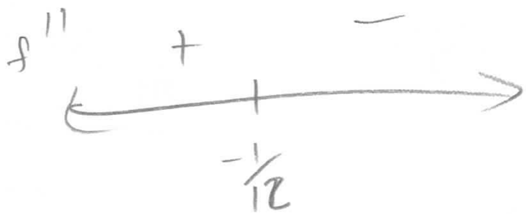
$f'(x) = -6x^2 - x + 1$



$= 0$ at $x = \frac{1 \pm \sqrt{1+24}}{-12}$

$= \frac{-1}{12} \pm \frac{5}{12} = \frac{-1}{2}, \frac{1}{3}$

$f''(x) = -12x - 1$

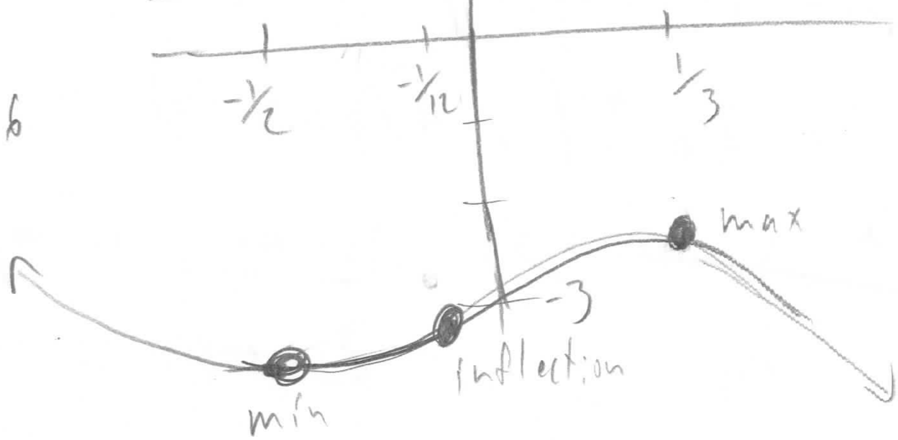


$= 0$ at $x = \frac{-1}{12}$

$f\left(-\frac{1}{2}\right) = -3.375$

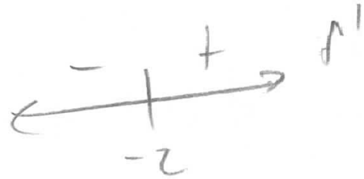
$f\left(\frac{1}{3}\right) = -2.796$

$f\left(-\frac{1}{12}\right) = -3.086$

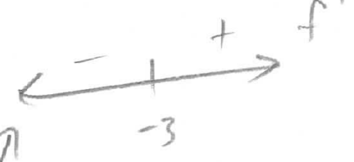


(53) $f(x) = xe^{2x}$

$f'(x) = x\,2e^{2x} + e^{2x} = e^{2x}(x+2)$



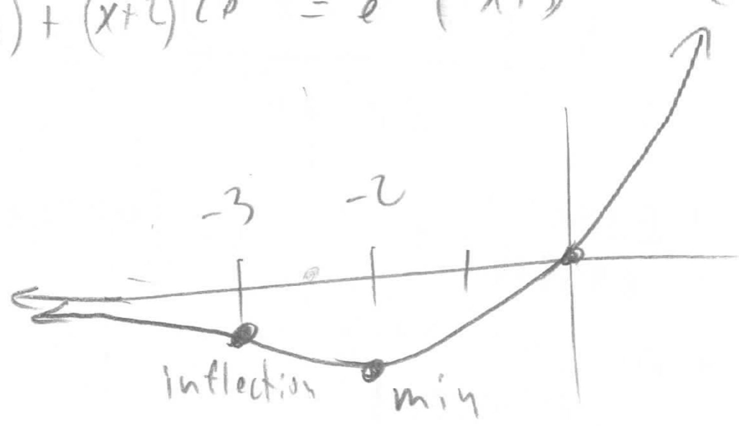
$f''(x) = e^{2x}(1) + (x+2)2e^{2x} = e^{2x}(x+3)$



$f(-2) = -2e^{-4}$

$f(-3) = -3e^{-6}$

$f(0) = 0$



ch 6

(11) $f(x) = -x^3+6x^2+1 \quad [-1, 6]$

$f'(x) = -3x^2+12x = 0 = -3x(x-4) \quad \begin{matrix} x=0 \\ x=4 \end{matrix}$

crit $\begin{cases} f(0) = 1 \\ f(4) = 33 \end{cases}$ - min of 1 at $x = 0, 6$

- max of 33 at $x = 4$

end $\begin{cases} f(-1) = 8 \\ f(6) = 1 \end{cases}$

(17) $f(x) = \dfrac{2\ln x}{x^2} \quad f'(x) = \dfrac{x^2 \frac{2}{x} - 2\ln x(2x)}{x^4}$

(a) $[1, 4]$:

$= \dfrac{2x - 4x\ln x}{x^4} = \dfrac{2(1-2\ln x)}{x^3}$

$f(1) = 0$ min

$f(4) = .17..$

$= 0$ at $x = e^{1/2} = \sqrt{e}$

$f(\sqrt{e}) = 1/e \approx .368\ldots$ max

(b) $[2, 5]$

$f(2) = .3466\ldots$ max

$f(5) = .1288\ldots$ ~~min~~

$\sqrt{e} \notin [2,5]$

(18) $f(x) = \dfrac{e^{2x}}{x^2}$ $\quad f'(x) = \dfrac{x^2(2e^{2x}) - e^{2x}(2x)}{x^4}$

$$= \frac{2xe^{2x}(x-1)}{x^4} = 0 \text{ at } x = 1$$

(a) $\left[\frac{1}{2}, 2\right]$

$f\left(\frac{1}{2}\right) = 4e \approx 10.9$

$f(2) = e^4/4 \approx 13.6$ max

$f(1) = e^2 \approx 7.4$ min

(b) $[1, 3]$

$f(1) = e^2 \approx 7.4$ min

$f(3) = e^6/9 \approx 44.8$ max

(48) 

Volume $= 40 = \pi r^2 h$ (constraint)

$$\text{Cost} = 4(\underbrace{\pi r^2}_{\text{top}} + \underbrace{\pi r^2}_{\text{bottom}}) + 3(\underbrace{2\pi r h}_{\text{side}})$$

$C(r) = 8\pi r^2 + 6\pi r\left(\dfrac{40}{\pi r^2}\right)$ $\quad$ (substitute $h = \dfrac{40}{\pi r^2}$)

$= 8\pi r^2 + \dfrac{240}{r}$

$C'(r) = 16\pi r - \dfrac{240}{r^2} = 0 \Rightarrow 16\pi r^3 = 240$

$$\begin{cases} r = \sqrt[3]{\dfrac{240}{16\pi}} \approx 1.684 \\ h \approx 4.490 \end{cases}$$

ch 7

(26) $\int \left(2x^{4/3} + x^{-1/2}\right) dx$

$$= \frac{2x^{7/3}}{(7/3)} + \frac{x^{1/2}}{1/2} + C = \frac{6}{7} x^{7/3} + 2x^{1/2} + C$$

(31)

$$\int x e^{3x^2} dx = \int \frac{1}{6} e^u du = \frac{e^u}{6} + C$$

$u = 3x^2$

$du = 6x\,dx$

$$= \frac{e^{3x^2}}{6} + C$$

(33)

$$\int \frac{3x}{x^2-1} dx = \int \frac{3}{2} \frac{1}{u} du = \frac{3}{2} \ln|u| + C$$

$u = x^2 - 1$

$du = 2x\,dx$

$$= \frac{3}{2} \ln|x^2-1| + C$$

(30) $$\int \frac{(3\ln x + 2)^4}{x} dx = \int \frac{u^4}{3} du = \frac{u^5}{15} + C$$

$$= \frac{(3\ln x+2)^5}{15} + C$$

$u = 3\ln x + 2$

$du = \frac{3}{x} dx$

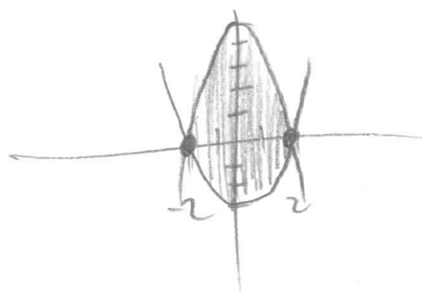
(52) $\int_0^2 x^2(3x^3+1)^{1/3}\,dx = \int_1^{25} \frac{1}{9}u^{1/3}\,du$

$u = 3x^3+1$
$du = 9x^2\,dx$

$= \frac{3}{4}\frac{1}{9}u^{4/3}\Big|_1^{25}$

$= \frac{25^{4/3}}{12} - \frac{1}{12} \approx 6$

(63) $f(x) = 5 - x^2,\ g(x) = x^2 - 3$



$5 - x^2 = x^2 - 3$
$8 = 2x^2$
$x = \pm 2$

$\int_{-2}^{2}[(5-x^2)-(x^2-3)]\,dx = \int_{-2}^{2}(8-2x^2)\,dx$

$= 8x - \frac{2x^3}{3}\Big|_{-2}^{2} = \left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right)$

$= 32 - \frac{32}{3} = 64/3$

Ch 9

29 $f(x,y) = 6x^2y^3 - 4y$

$f_x = 12xy^3, \quad f_y = 18x^2y^2 - 4$

32 $f(x,y) = \dfrac{2x+5y^2}{3x^2+y^2} \qquad f_x = \dfrac{(3x^2+y^2)(2) - (2x+5y^2)(6x)}{(3x^2+y^2)^2}$

$f_y = \dfrac{(3x^2+y^2)(10y) - (2x+5y^2)(2y)}{(3x^2+y^2)^2}$

48 $z = x^3 - 8y^2 + 6xy + 4$

$z_x = 3x^2 + 6y \qquad z_y = -16y + 6x$

$z_x = 0 \Rightarrow y = \dfrac{-x^2}{2}, \quad z_y = 0 \Rightarrow y = \dfrac{3x}{8}$

$\dfrac{3x}{8} = \dfrac{-x^2}{2} \Rightarrow x^2 + \dfrac{3}{4}x = 0$

$x\left(x + \dfrac{3}{4}\right) = 0$

$x = 0, -3/4$

$\downarrow \quad \downarrow$

$y = 0, -9/32$

$\left.\begin{array}{l}(0,0)\\ \left(-\frac{3}{4}, -\frac{9}{32}\right)\end{array}\right\}$ crit pts

$D_z(x,y) = (6x)(-16) - 6^2$

$= -96x - 36$

- $D_z(0,0) = -36 < 0$ saddle at $(0,0)$
- $D_z\left(\frac{-3}{4}, \frac{-9}{32}\right) > 0, \quad z_{xx}\left(\frac{-3}{4}, \frac{-9}{32}\right) = \frac{-18}{4} < 0$ max at $\left(-\frac{3}{4}, -\frac{9}{32}\right)$

51 $z = x^3 + y^2 + 2xy - 4x - 3y - 2$

$$\left.\begin{aligned} z_x &= 3x^2 + 2y - 4 = 0 \\ z_y &= 2y + 2x - 3 = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} y &= -\tfrac{3}{2}x^2 + 2 \\ x &= -y + \tfrac{3}{2} \end{aligned}$$

$x = \frac{3}{2}x^2 - 2 + \frac{3}{2}$, $0 = 3x^2 - 2x - 1$, $x = 1, -\frac{1}{3}$

$\downarrow \quad \downarrow$

$y = \frac{1}{2} \quad \frac{11}{6}$.

$(1, \frac{1}{2}), (-\frac{1}{3}, \frac{11}{6})$ crit pts

$D_z(x,y) = (6x)(2) - 2^2 = 12x - 4$

- $D_z(1, \frac{1}{2}) > 0$, $z_{xx}(1, \frac{1}{2}) > 0$, min at $(1, \frac{1}{2})$
- $D(-\frac{1}{3}, \frac{11}{6}) < 0$ saddle at $(-\frac{1}{3}, \frac{11}{6})$