

1. $\int 4t\sqrt{8-t} dt =$

Let $u = 8 - t$. Then $t = 8 - u$ and $du = -dt$. Substituting, we get

$$\begin{aligned} \int 4t\sqrt{8-t} dt &= \int -4(8-u)\sqrt{u} du \\ &= \int (-32u^{1/2} + 4u^{3/2}) du \\ &= \frac{-64}{3}u^{3/2} + \frac{8}{5}u^{5/2} \\ &= \frac{-64}{3}(8-t)^{3/2} + \frac{8}{5}(8-t)^{5/2}. \end{aligned}$$

2. $\int \frac{y^2 + y}{(2y^3 + 3y^2 + 1)^{2/3}} dy =$

Let $u = 2y^3 + 3y^2 + 1$, $du = (6y^2 + 6y)dy$. Then

$$\begin{aligned} \int \frac{y^2 + y}{(2y^3 + 3y^2 + 1)^{2/3}} dy &= \int \frac{u^{-2/3}}{6} du \\ &= \frac{u^{1/3}}{2} \\ &= \frac{(2y^3 + 3y^2 + 1)^{1/3}}{2} \end{aligned}$$

3. $\int_1^e \frac{\sqrt{\ln x}}{x} dx =$

Let $u = \ln x$, $du = 1/x$, $u(1) = 0$, $u(e) = 1$. Then

$$\begin{aligned} \int_1^e \frac{\sqrt{\ln x}}{x} dx &= \int_0^1 u^{1/2} du \\ &= \frac{2u^{3/2}}{3} \Big|_0^1 \\ &= 2/3. \end{aligned}$$

4. $\int_{-2}^3 (-x^2 - 3x + 5) dx =$

$$\left(\frac{-x^3}{3} - \frac{3x^2}{2} + 5x \right) \Big|_{-2}^3 = 35/6$$

5. Estimate the definite integral in (4) by partitioning $[-2, 3]$ into five intervals of equal length and using the value of the function at the left endpoint of each interval to approximate the function on that interval. (Your approximation won't necessarily be "close" to the actual value from (4).)

The left endpoints of the interval are $x = -2, -1, 0, 1, 2$ with corresponding function values $f(-2) = 7$, $f(-1) = 7$, $f(0) = 5$, $f(1) = 1$, $f(2) = -5$. So

$$\sum_i f(x_i) \Delta x = 7 + 7 + 5 + 1 - 5 = 15.$$