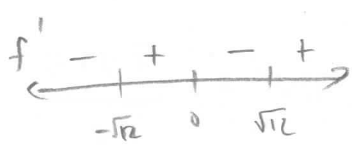
Math 1081, Quiz 5 Name: ______________________

(You may use a calculator or leave any numerical answers in terms of $\sqrt{\ }, \pi$, etc. Feel free to write on the reverse if necessary.)

1. Consider the function
$$f(x) = \frac{x^4}{4} - 6x^2 + 1.$$
   (a) Find f'.
   (b) What are the critical numbers of f?
   (c) On what intervals is f increasing/decreasing?
   (d) List and identify (as maximum/minimum) any local extrema.
   (e) Find f''.
   (f) What are the critical numbers of f'?
   (g) On what intervals is f concave up/concave down?
   (h) List any inflection points.
   (i) Use the above information to sketch a graph of f, labeling local extrema and inflection points. (Don't just copy a graph from a calculator.)

(a) $f' = x^3 - 12x = x(x-\sqrt{12})(x+\sqrt{12})$

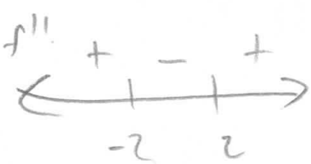
(b) $x = 0, \pm\sqrt{12}$

(c) 

increasing on $(-\sqrt{12}, 0), (\sqrt{12}, \infty)$
decreasing on $(-\infty, -\sqrt{12}), (0, \sqrt{12})$

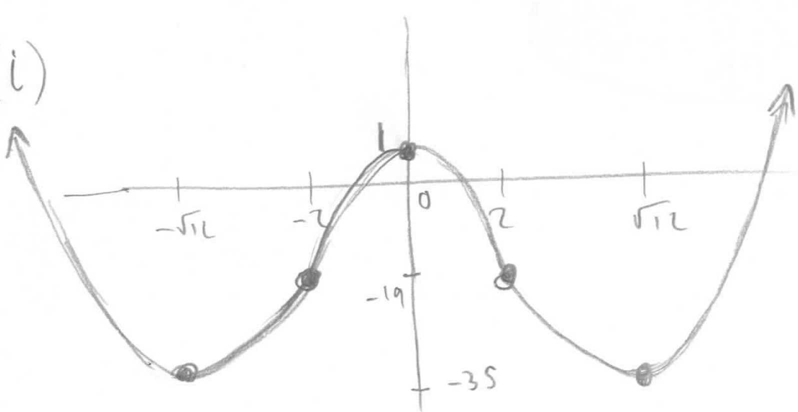
(d) local min of $36 - 72 + 1 = -35$ at $x = -\sqrt{12}$
local max of 1 at $x = 0$
local min of -35 at $x = \sqrt{12}$

(e) $f'' = 3x^2 - 12 = 3(x-2)(x+2)$ (f) $x = \pm 2$

(g) 

CCU on $(-\infty, -2), (2, \infty)$
CCD on $(-2, 2)$

(h) inflection pts $(\pm 2, -19)$

(i) 

2. Suppose a cylindrical cup has volume 1000 cm^3 (no top). Minimize the surface area of the cup as follows:

   (a) Write an equation for the surface area of the cup (circular bottom of radius r and a (curved) rectangular side of height h and width $2\pi r$).

   (b) Write an equation (relating r and h) for the constraint that the volume is 1000 cm^3.

   (c) Solve the equation from (b) for h.

   (d) Use the expression from (c) to write the surface area as a function of the variable r.

   (e) Assuming that you obtained $S(r) = \pi r^2 + \dfrac{2000}{r}$ for part (d), find the value of r that minimizes the surface area S.

(a) $\pi r^2 + 2\pi r h$

(b) $\pi r^2 h = 1000$

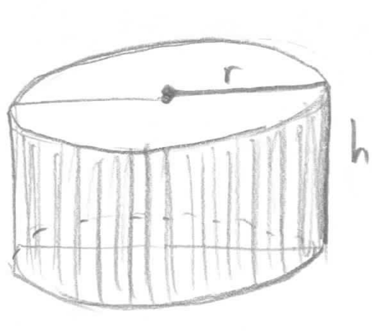
(c) $h = \dfrac{1000}{\pi r^2}$

(d) $\pi r^2 + 2\pi r \left(\dfrac{1000}{\pi r^2}\right) = \pi r^2 + \dfrac{2000}{r} = S(r)$

(e) $S'(r) = 2\pi r - \dfrac{2000}{r^2} = 0$, $r = \sqrt[3]{\dfrac{1000}{\pi}} = \dfrac{10}{\sqrt[3]{\pi}}$

≈ 6.83 cm

($h = \dfrac{10}{\sqrt[3]{\pi}}$ as well, cup is twice as wide as it is tall)



$r = h = \dfrac{10}{\sqrt[3]{\pi}}$