Math 1081, Quiz 5 Name: _____

(You may use a calculator or leave any numerical answers in terms of $\sqrt{}, \pi$, etc. Feel free to write on the reverse if necessary.)

1. Consider the function
$$f(x) = \frac{x^4}{4} - 6x^2 + 1.$$
   (a) Find f'.
   (b) What are the critical numbers of f?
   (c) On what intervals is f increasing/decreasing?
   (d) List and identify (as maximum/minimum) any local extrema.
   (e) Find f''.
   (f) What are the critical numbers of f'?
   (g) On what intervals is f concave up/concave down?
   (h) List any inflection points.
   (i) Use the above information to sketch a graph of f, labeling local extrema and inflection points. (Don't just copy a graph from a calculator.)

2. Suppose a cylindrical cup has volume 1000 cm^3 (no top). Minimize the surface area of the cup as follows:

   (a) Write an equation for the surface area of the cup (circular bottom of radius r and a (curved) rectangular side of height h and width $2\pi r$).

   (b) Write an equation (relating r and h) for the constraint that the volume is 1000 cm^3.

   (c) Solve the equation from (b) for h.

   (d) Use the expression from (c) to write the surface area as a function of the variable r.

   (e) Assuming that you obtained $S(r) = \pi r^2 + \dfrac{2000}{r}$ for part (d), find the value of r that minimizes the surface area S.