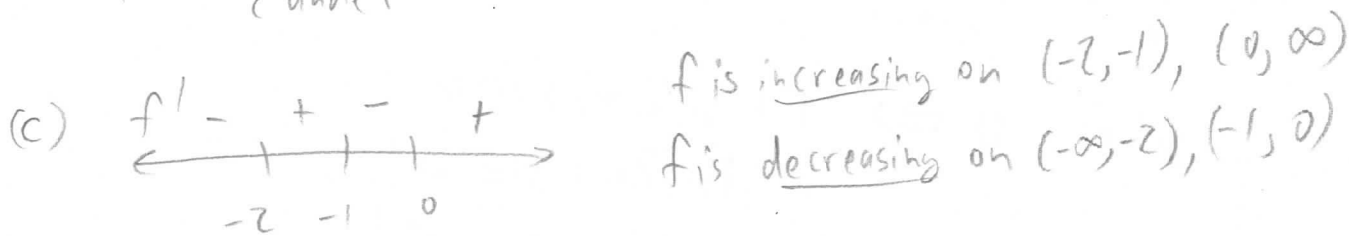


1. Let $f(x) = x^4 + 4x^3 + 4x^2 + 1$.

- 1 (a) What is $f'(x)$?
- 3 (b) What are the critical numbers of f ?
- 2 (c) On what intervals is f increasing/decreasing?
- 3 (d) Locate and identify any local extrema of f .

(a) $f'(x) = 4x^3 + 12x^2 + 8x = 4x(x^2 + 3x + 2)$
 $= 4x(x+2)(x+1)$

(b) $f'(x) = \begin{cases} 0 \\ \text{undef} \end{cases}$ at $x = 0, -1, -2$

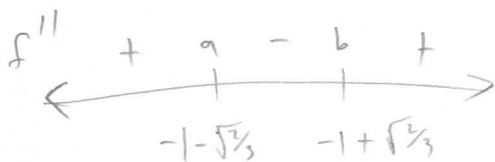


(d) • f has a local minimum of $f(-2) = 16 - 32 + 16 + 1 = 1$ at $x = -2$

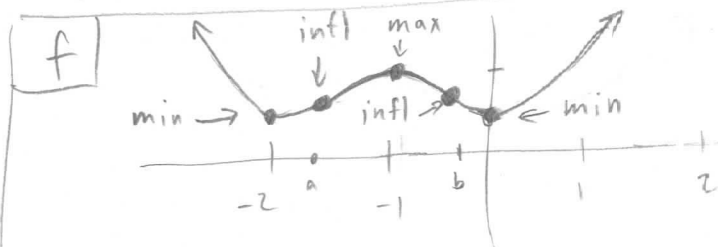
• f has a local maximum of $f(-1) = 1 - 4 + 4 + 1 = 2$ at $x = -1$

• f has a local minimum of $f(0) = 1$ at $x = 0$

$f''(x) = 12x^2 + 24x + 8$
 $= 12(x^2 + 2x + \frac{2}{3})$
 $= 0$ at $x = \frac{-2 \pm \sqrt{4 - \frac{4}{3}}}{2}$
 $= -1 \pm \sqrt{\frac{2}{3}}$



f is CCU on $(-\infty, a), (b, \infty)$
 f is CCD on (a, b)
 f has inflection points at $(a, f(a)), (b, f(b))$



2. Let $g(x) = (2x + 3) \ln(2x + 3)$.

(a) What is the domain of g ?

(b) What is $g'(x)$?

(c) What are the critical numbers of g ? (You should get $x = \frac{e^{-1} - 3}{2}$.)

(d) On what intervals is g increasing/decreasing?

(e) Locate and identify any local extrema of g .

(a) $(-\frac{3}{2}, \infty)$

(b) $(2x+3) \frac{2}{2x+3} + 2 \ln(2x+3) = 2(-1 + \ln(2x+3)) = g'(x)$

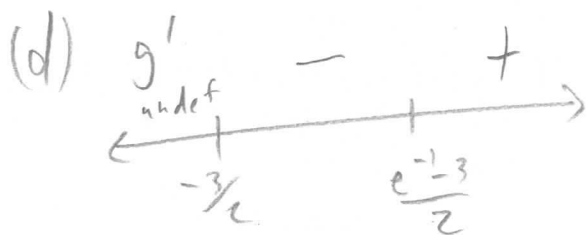
(c) $g'(x) = \begin{cases} 0 \\ \text{undef} \end{cases}$ (domain of g' is $(-\frac{3}{2}, \infty)$)

$$g'(x) = 0 \Rightarrow 1 + \ln(2x+3) = 0$$

$$\ln(2x+3) = -1$$

$$2x+3 = e^{-1}$$

$$x = \frac{e^{-1} - 3}{2}$$



- g is increasing on $(\frac{e^{-1}-3}{2}, \infty)$
- g is decreasing on $(-\frac{3}{2}, \frac{e^{-1}-3}{2})$

(e) g has a local minimum of $g(\frac{e^{-1}-3}{2}) = e^{-1} \ln e^{-1} = \frac{-1}{e}$
 at $x = \frac{e^{-1}-3}{2}$