

Robert Hines

1. Suppose the height of a ball thrown out of a window (in feet above the ground after t seconds) is given by

$$h(t) = -16t^2 + 48t + 64, \text{ for } 0 \leq t \leq t_f,$$

(where t_f is the time at which the ball hits the ground, $h(t_f) = 0$).

- What is the initial height?
- What is the initial velocity?
- When does the ball reach its maximum height?
- What is the maximum height?
- When does the ball hit the ground?
- With what velocity does the ball hit the ground?

(a) $h(0) = 64 \text{ ft}$

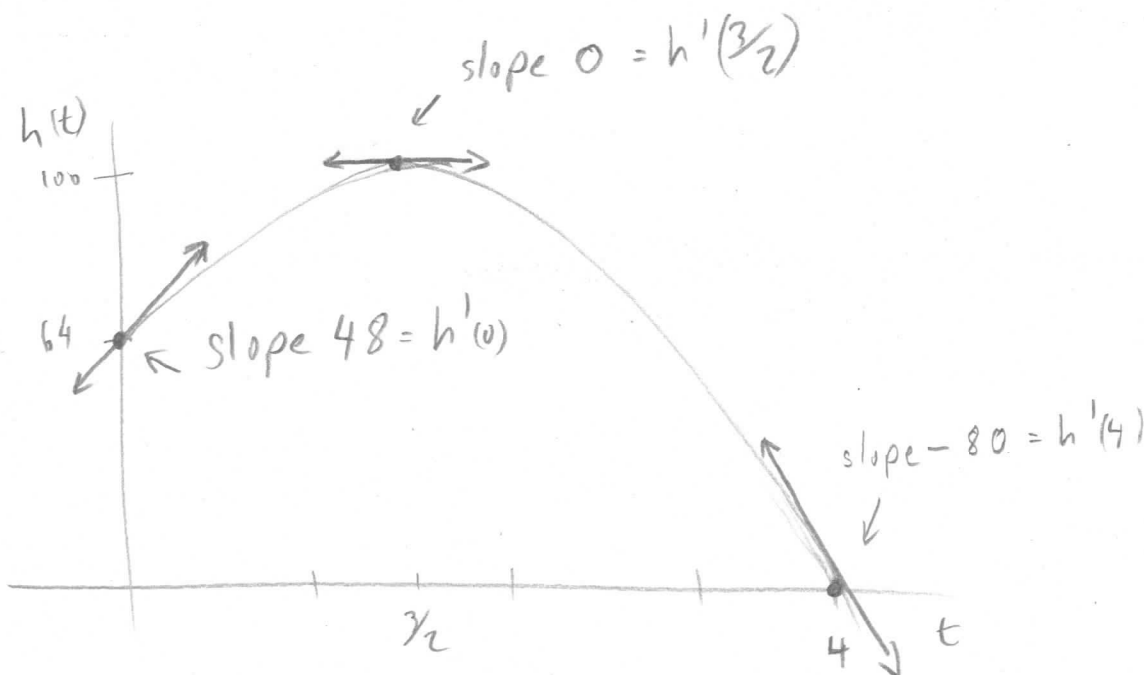
(b) $h'(0) = 48 \text{ ft/sec}$

(c) $h'(t) = 0 = -32t + 48, t = \frac{-48}{-32} = \frac{3}{2} \text{ sec.}$

(d) $h\left(\frac{3}{2}\right) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 64 = -36 + 72 + 64 = 100 \text{ ft}$

(e) $h(t) = 0 = -16(t^2 - 3t - 4) = -16(t-4)(t+1), t_f = 4 \text{ sec.}$

(f) $h'(4) = -32(4) + 48 = -80 \text{ ft/sec}$



3. Find the following derivatives:

(a) $\frac{d}{dx} \frac{e^{3x}}{x^3 + 1}$

(b) $\frac{d}{dx} x^2 \ln(3x - 1)$

(c) $\frac{d}{dx} \sqrt[4]{4^x + x^4}$

$$(a) \frac{(x^3+1)(3e^{3x}) - (e^{3x})(3x^2)}{(x^3+1)^2} = \frac{3e^{3x}(x^3 - x^2 + 1)}{(x^3+1)^2}$$

$$(b) x^2 \frac{3}{3x-1} + \ln(3x-1) 2x = \frac{3x^2}{3x-1} + 2x \ln(3x-1)$$

$$(c) \frac{1}{4} (4^x + x^4)^{-3/4} (\ln 4 \cdot 4^x + 4x^3)$$

$$= \frac{x^3 + (\ln 4)4^{x-1}}{\sqrt[4]{(4^x + x^4)^3}}$$