Math 1081, Quiz 2 Name: Robert Hines

1. Using the definition of the derivative as a limit, find $f'(x)$ if $f(x) = \frac{2}{x-2}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{h}\left(\frac{2}{x+h-2} - \frac{2}{x-2}\right)$$

$$= \lim_{h \to 0} \frac{2(x-2) - 2(x+h-2)}{h(x+h-2)(x-2)}$$

$$= \lim_{h \to 0} \frac{\cancel{2x} \cancel{-4} \cancel{-2x} - 2h \cancel{+4}}{h(x+h-2)(x-2)}$$

$$= \lim_{h \to 0} \frac{-2h}{h(x+h-2)(x-2)} = \lim_{h \to 0} \frac{-2}{(x+h-2)(x-2)}$$

$$= \frac{-2}{(x-2)^2}$$

2. Find the derivative of

$$y = \frac{3x^2 - 2}{\sqrt{x}} + \frac{x}{5\sqrt[3]{x^2}}$$

(Hint: simplify y so that it involves only power functions, multiplication by constants, and addition/subtraction - then differentiate.)

$$y = \frac{3x^2}{x^{1/2}} - \frac{2}{x^{1/2}} + \frac{1}{5}\frac{x}{x^{2/3}}$$

$$= 3x^{(2-1/2)} - 2x^{-1/2} + \frac{1}{5}x^{(1-2/3)}$$

$$= 3x^{3/2} - 2x^{-1/2} + \frac{1}{5}x^{1/3}$$

$$y' = 3\left(\frac{3}{2}x^{1/2}\right) - 2\left(-\frac{1}{2}x^{-3/2}\right) + \frac{1}{5}\left(\frac{1}{3}x^{-2/3}\right)$$

$$\left(\frac{d}{dx}(x^n) = nx^{n-1}\right)$$

$$= \frac{9}{2}x^{1/2} + x^{-3/2} + \frac{1}{15}x^{-2/3}$$

$$= \frac{9\sqrt{x}}{2} + \frac{1}{\sqrt{x^3}} + \frac{1}{15\sqrt[3]{x^2}}$$