Math 1081, Quiz 1 Name: Robert Hines

1. Suppose $\log_b x = 2, \log_b y = 3, \log_b z = 5$. Find the following:

   (a) $\log_b \left(\frac{\sqrt{xyb}}{zb^2}\right)$ $= -4$

   (b) $\frac{x^3y^2}{\sqrt{bz}}$ if $b = 2$.

(a) $\log_b \frac{\sqrt{xyb}}{zb^2} = \frac{1}{2}(\log_b x + \log_b y + \log_b b) - \log_b z - 2\log_b b = 3-5-2=-4$

(b) $b^2 = x, b^3 = y, b^5 = z$

$$\frac{x^3y^2}{\sqrt{bz}} = \frac{b^6 \cdot b^6}{\sqrt{b^6}} = b^9 = 512 \text{ if } b=2$$

2. Suppose a 5-year CD (cash deposit) has an interest rate of 2.27% compounded daily. With a deposit of P_0 dollars, the account grows like

$$P(t) = P_0(1 + .0227/365)^{365t}.$$

(You may leave your answers in terms of exp/log if you don't want to use a calculator.)

   (a) What is the value of the account (after 5 years) if \$10,000 was deposited initially?

   (b) How long does it take for the value of the account to double?

(a) $P(5) = 10000\left(1+\frac{.0227}{365}\right)^{365 \cdot 5} \approx \$11201.88$

(b) $P(t) = 2P(0)$

$$\cancel{P_0}\left(1+\frac{.0227}{365}\right)^{365t} = 2\cancel{P_0}$$

$$365t\ln\left(1+\frac{.0227}{365}\right) = \ln 2$$

$$t = \ln 2 / 365 \cdot \ln\left(1+\frac{.0227}{365}\right) \approx 30.54 \text{ years}$$

3. Consider

$$g(x) = \frac{2x^2 - 8}{x^2 - 5x + 6}.$$

(a) Where is g discontinuous?

(b) Find $\lim_{x \to 2} g(x)$.

(c) Find $\lim_{x \to 3^-} f(x)$, $\lim_{x \to -3^+} g(x)$.

(d) Find $\lim_{x \to \infty} g(x)$.

$$g(x) = \frac{2(x-2)(x+2)}{(x-2)(x-3)} = \begin{cases} \frac{2x+4}{x-3} & x \neq 2 \\ \text{undef} & x = 2 \end{cases}$$

(a) discontinuous at $x = 2, 3$

(b) $\lim_{x \to 2} g(x) = \lim_{x \to 2} \frac{2(x+2)}{x-3} = -8$

(c) $\lim_{x \to 3^+} g(x) = +\infty \quad \left(\frac{2x+4}{x-3} > 0 \text{ for } x > 3 \right)$

$\lim_{x \to 3^-} g(x) = -\infty \quad \left(\frac{2x-4}{x-3} < 0 \text{ for } x < 3 \right)$

(d) $\lim_{x \to \infty} g(x) = \lim_{x \to \infty} \frac{2 + 4/x}{1 - 3/x} = \frac{2+0}{1-0} = 2$

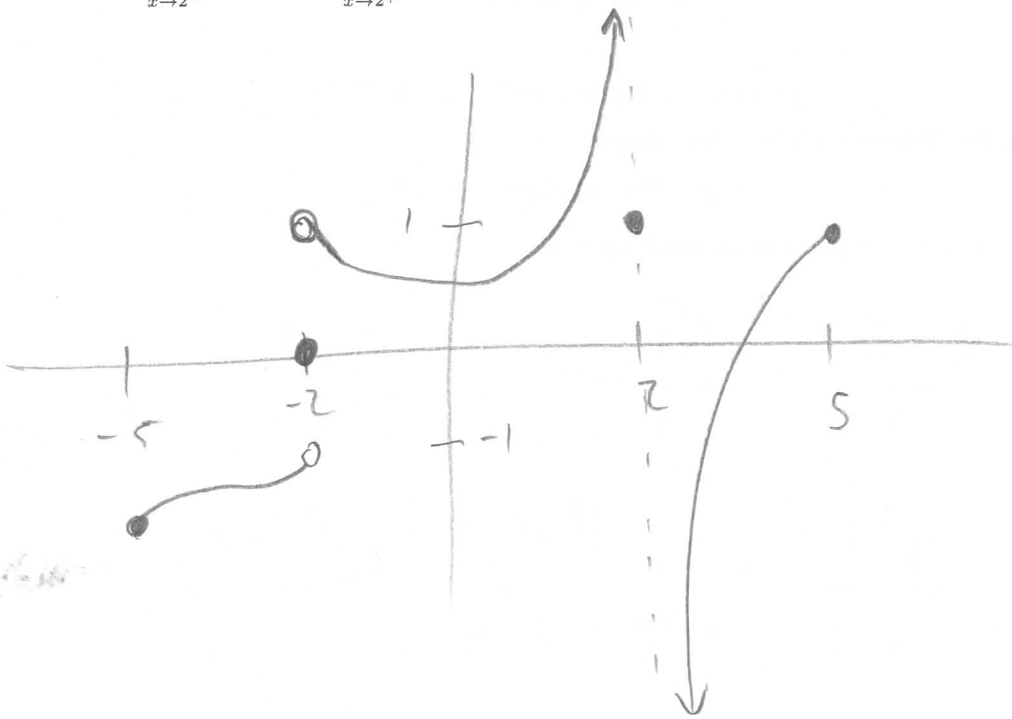
4. Draw the graph of a function f with the following properties:

(a) f has domain $[-5, 5]$

(b) f is continuous except at $x = -2, 2$

(c) $\lim_{x \to -2^-} f(x) = -1$, $\lim_{x \to -2^+} f(x) = 1, f(-2) = 0$

(d) $\lim_{x \to 2^-} f(x) = +\infty$, $\lim_{x \to 2^+} f(x) = -\infty, f(2) = 1$



5. Consider the function $f(x) = \sqrt[3]{x}$ $(= x^{1/3})$.

(a) What is the average rate of change of f over the interval $[1, 8]$?

(b) Find the instantaneous rate of change of f at $x = 1$.
(Hint: Multiply both the numerator and denominator of

$$\frac{x^{1/3} - 1}{x - 1} \text{ or } \frac{(1+h)^{1/3} - 1}{h}$$

by

$$x^{2/3} + x^{1/3} + 1 \text{ or } (1+h)^{2/3} + (1+h)^{1/3} + 1,$$

i.e. use the "difference of cubes" factorization

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

to get rid of the $x - 1$ or h in the denominator.)

(a) $$\frac{f(8) - f(1)}{8 - 1} = \frac{\sqrt[3]{8} - \sqrt[3]{1}}{8 - 1} = \frac{2 - 1}{7} = \frac{1}{7}$$

(b) $$\lim_{x \to 1} \frac{x^{1/3} - 1}{x - 1} = \lim_{x \to 1} \frac{(x^{1/3} - 1)}{x - 1} \frac{x^{2/3} + x^{1/3} + 1}{x^{2/3} + x^{1/3} + 1}$$

$$= \lim_{x \to 1} \frac{x - 1}{(x - 1)(x^{2/3} + x^{1/3} + 1)} = \lim_{x \to 1} \frac{1}{x^{2/3} + x^{1/3} + 1}$$

$$= \frac{1}{1 + 1 + 1} = \frac{1}{3}$$