MATH 8174: Assignment 9

1. Let L be a nonabelian Lie algebra of dimension 2 over a field k. With respect to a suitable basis of L, find the matrix representation of the adjoint representation of L.

2. Let $\mathrm{Der}(A)$ be the set of derivations of an algebra A over k. Show that $\mathrm{Der}(A)$ is a subalgebra of the Lie algebra $\mathrm{End}_k(A)$ equipped with the usual bracket.

3. Prove Leibniz's Rule: if D is a derivation of an algebra A and $x, y \in A$, then

$$D^n(xy) = \sum_{i=0}^{n} \binom{n}{r} (D^r(x))(D^{n-r}(y)).$$

4. Recall that $\mathrm{ad}(L) = \{\mathrm{ad}(x) : x \in L\}$. Show that $\mathrm{ad}(L)$ is a Lie algebra under the usual Lie bracket on endomorphisms. Show furthermore that $\mathrm{ad}(L)$ is an ideal of the Lie algebra $\mathrm{Der}(L)$ as defined in Question 2.

5. Let x and l be elements of a Lie algebra of endomorphisms $L = \mathrm{End}_k(V)$, equipped with the usual Lie bracket. State and prove a formula for the coefficients $c_k(t)$ in the equation

$$\mathrm{ad}(x)^t = \sum_{k=0}^{t} c_k(t) x^{t-k} l x^k,$$

where the product on the right is the usual associative product of endomorphisms.