MATH 8174: Assignment 7

1. Recall that the conjugacy classes of the symmetric group S_n are in natural bijection with the partitions of n. Find the order of the centralizer of a typical element of such a conjugacy class. State and prove a theorem that determines when the centralizer of such an element lies entirely within the alternating group A_n, in terms of the associated partition. What are the conjugacy classes of the alternating group A_n?

2. Find the character table of the alternating group A_6, and then use that to find the character table of the symmetric group S_6. (You may use the character tables of A_5 and S_5 without proof; try inducing characters from these latter two groups.)

3. Denote by $a(G)$ the complex representation ring of G. If V is a complex representation, write $\Psi^2(V)$ for the element $[S^2(V)] - [\Lambda^2(V)] \in a(G)$. Show that we have:

(i) $\Psi^2(V \oplus W) = \Psi^2(V) + \Psi^2(W)$, and

(ii) $\Psi^2(V \otimes W) = \Psi^2(V) \times \Psi^2(W)$.

Deduce that Ψ^2 extends to a ring homomorphism $\Psi^2 : a(G) \to a(G)$. Composition with Ψ^2 takes a ring homomorphism $\alpha : a(G) \to \mathbb{C}$ to $(\alpha \circ \Psi^2) : a(G) \to \mathbb{C}$. Prove that Ψ^2 induces a map on the set of conjugacy classes of G, and describe this map in terms of elements of G.