MATH 6130: Second midterm examination. Wednesday, 1 November 2023.

Put **your name** on each answer sheet. Answer **both** questions.

Show all your work.

Formula sheets, calculators, notes and books are not permitted.

1.

(i) Show how to construct a nonabelian group of order 12 having an element of order 4.

(ii) Prove that the group from (i) is not isomorphic to any of the alternating or dihedral groups. [You should provide justification for any results you use about element orders in specific groups.]

2. [In this question, you may use any standard results from class or the book, so long as it is clear how you are using them, but you may not use methods from character theory.]

(i) Explain why a nonabelian group of order 39 exists, and let G be such a group.

(ii) How many elements does G have of (a) order 1, (b) order 3, (c) order 13, and (d) order 39?

(iii) Find the order of the center, $Z(G)$, of G.

(iv) Find the order of the commutator subgroup, G', of G.

(v) Determine whether G is (a) nilpotent and/or (b) solvable.