1. (20) Let $\mathcal{P}_n$ denote the $(n+1)$-dimensional real vector space consisting of polynomials of degree at most n (together with the zero polynomial) under the usual operations of addition and scalar multiplication. Let $T : \mathcal{P}_2 \to \mathcal{P}_3$ be the transformation that maps a polynomial $\mathbf{p}(t)$ to the polynomial $(t+5)\mathbf{p}(t)$.

(i) Find the image of $\mathbf{p}(t) = 2 - t + t^2$.

(ii) Show that T is a linear transformation.

(iii) Find the matrix for T relative to the bases $\{1, t, t^2\}$ and $\{1, t, t^2, t^3\}$.

2. (20) Let $\mathcal{P}_2$ be the set of all polynomials of the form $a_0+a_1t+a_2t^2$, where $a_0, a_1, a_2 \in \mathbb{R}$. Recall that $\mathcal{P}_2$ becomes a vector space under the usual operations of addition and scalar multiplication of functions. Define $W = \{\mathbf{p}(t) \in \mathcal{P}_2 : \mathbf{p}(-1) = 0\}$.

(i) Show that two of the following polynomials lie in W, and two do not:

$$-1, \quad t+1, \quad t-1, \quad t^2-1.$$

(ii) Prove (carefully!) that W is a subspace of $\mathcal{P}_2$.

(iii) Write down two linearly independent elements of W and explain without calculation why they must form a basis of W.

3. (20) Let C be the matrix

$$\begin{bmatrix} 1/\sqrt{18} & 1/\sqrt{2} & -2/3 \\ 4/\sqrt{18} & 0 & 1/3 \\ 1/\sqrt{18} & -1/\sqrt{2} & -2/3 \end{bmatrix}$$

(i) Prove that the columns of C form an orthonormal set of vectors in $\mathbb{R}^3$.

(ii) Without further calculation, write down the inverse of C.

(iii) Explain why the above results imply that either $\det C = 1$ or $\det C = -1$. (You may use any standard results about determinants if it is clear how you are using them.)

(iv) Calculate $\det C$.

4. (20) Calculate (a) the classical adjoint (adjugate) matrix adj A and (b) the inverse matrix A^{-1}, where

$$A = \begin{bmatrix} 0 & -2 & -1 \\ 3 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}.$$

5. (20) Let A be the matrix

$$\begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}.$$

(i) Find the eigenvalues of A.

(ii) Find a basis for each eigenspace of A (as a real vector space).

(iii) Diagonalize A, or explain why it is not diagonalizable.

Name: ______________________________

University of Colorado

Mathematics 3130: Final Exam

December 2, 2011

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	