University of Colorado
Department of Mathematics

2005/2006 Semester 1 | Math 2400 Calc 3 | Discussion and practice, final exam

There is no need to try all the problems listed. I was just asked to list some problems from the book that might be helpful. The HW problems are the most helpful.

1. Material from book needed for exam:
   (a) Material covered in review sheets for Midterms 1, 2, and 3 (check relevant review sheets on web). This includes Chapters 11, 12, 13.1 – 13.7. Look over midterm exam problems you did badly on.
   (b) Sections 13.8, Surface area: parametric surfaces in $\mathbb{R}^3$, computing normal vectors to a parametric surface, special case where the surface is the graph of a function, sufrace area of parametric surfaces, surface area in rectangular coordinates, surface area in cylindrical coordinates.
   Review homework and try problems pp. 1002-3, 3, 5, 9, 11, 15.
   (c) Section 13.9, Change of variables in multiple integrals: change of variables given by differentiable tranformations T, the inverse transformation T^{-1}, change of variables in double integrals, the Jacobian $J_T(u, v) = \frac{\partial(x,y)}{\partial(u,v)}$ of a continuously differentiable transformation $T : \mathbb{R}^2_{uv} \to \mathbb{R}^2_{xy}$, change of variable formula for a continuously differentiable transformation T

$$\int\int_R F(x, y)dxdy \ = \ \int\int_S F(T(u, v))|J_T(u, v)|dudv,$$

   special case of polar coordinates, change of variable in triple integrals, the Jacobian $J_T(u, v, w) = \frac{\partial(x,y,z)}{\partial(u,v,w)}$ of a continuously differentiable transformation $T : \mathbb{R}^3_{uvw} \to \mathbb{R}^3_{xyz}$, change of variable formula

$$\int\int\int_R F(x, y, z)dxdydz \ = \ \int\int\int_S F(T(u, v, w))|J_T(u, v, w)|dudvdw.$$

   Special case of spherical coordinates.
   Review hw and try problems pp. 1009-1011, 3, 9, 10, 11, 17.
   (d) Section 14.1, Vector fields: definition and examples of vector fields in $\mathbb{R}^2$ and $\mathbb{R}^3$, the gradient vector field ∇f of a scalar function f, basic vector field operations: the divergence div$\mathbf{F} = \nabla \cdot \mathbf{F}$ of a vector field $\mathbf{F}$, the curl curl$\mathbf{F} = \nabla \times \mathbf{F}$ of a vector field $\mathbf{F}$, the identities curl(grad$f$) $= \nabla \times \nabla f = \mathbf{0}$, and other vector operation identities.
   Review hw and try problems pp. 1020-1021, 9, 15, 19, 23, 25, 34, 35.

(e) Section 14.2, Line integrals: line integral of a function along a smooth curve $\int_{\mathbf{C}} f(x,y)ds$ (in $\mathbb{R}^2$) and $\int_{\mathbf{C}} f(x,y,z)ds$ (in $\mathbb{R}^3$), definition, examples and formulas, relationship to line integral of f with respect to arc length, line integrals with respect to coordinate variables - definition and examples, the line integral $\int_{\mathbf{C}} \mathbf{F}\cdot\mathbf{T}ds$, recall in $\mathbb{R}^3$ if $\mathbf{F} = \langle P, Q, R\rangle$, we have

$$\int_{\mathbf{C}} \mathbf{F}\cdot\mathbf{T}ds = \int_{\mathbf{C}} Pdx + Qdy + Rdz,$$

(and a similar formula for $\mathbf{F}$ and $\mathbf{C}$ in $\mathbb{R}^2$) work and line integrals.
Review hw (assignments 30 and 31) and try problems pp. 1030-1031, 1, 3, 6, 15, 17, 35.

(f) Section 14.3, the Fundamental Theorem and independence of path: the Fundamental Theorem for Line Integrals: under appropriate conditions on f and $\mathbf{C}$, $\int_{\mathbf{C}} \nabla f\cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$, Independence of Path for line integrals of vector fields, conservative vector fields and independence of path, Theorem: $\int_{\mathbf{C}} \mathbf{F}\cdot\mathbf{T}ds$ is independent of the path in a plane or space region D if and only if $\mathbf{F} = \nabla f$ for some function f defined on D, Conservative vector fields and potential functions, calculating a potential function for a conservative vector field, Theorem: a continuously differentiable vector field $\mathbf{F} = \langle P, Q\rangle$ defined on a rectangle $\mathcal{R}$ is conservative in $\mathcal{R}$ if and only if $P_y = Q_x$ at each point of $\mathcal{R}$, conservative force fields and conservation of energy.
Review hw and try problems pp. 1038-1039, 5, 7, 9, 25, 33.

(g) Section 14.4 : Green's Theorem: If $\mathcal{C}$ is a positively orineted piecewise smooth simple closed curve bounding the region $\mathcal{R}$ in $\mathbb{R}^2$, and if $\mathbf{F} = \langle P, Q\rangle$ is a continuously differentiable vector field on $\mathcal{R}$, then $\int_{\mathbf{C}} Pdx + Qdy = \int\int_{\mathcal{R}}(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}dA$. Examples and applications of Green's Theorem, use of Green's Theorem to evaluate areas, divergence and flux of a vector field, the flux Φ of $\mathbf{F}$ across $\mathbf{C}$ defined by $\Phi = \int_{\mathbf{C}} \mathbf{F}\cdot\mathbf{n}ds$, the vector form of Green's theorem: under appropriate hypotheses $\int_{\mathbf{C}} \mathbf{F}\cdot\mathbf{n}dx = \int\int_{\mathcal{R}} \nabla\cdot\mathbf{F}dA$, examples.
Review hw and try problems pp. 1047-1048, 3, 5, 9, 13, 17, 19, 20, 24.

(h) Section 14.5 : Surface Integrals: surface integral of a scalar function f over the surface S, formula $\int\int_S f(x,y,z)dS = \int\int_D f(\mathbf{r}(u,v))|\mathbf{N}(u,v)|dudv$, formula for normal vector $\mathbf{N} = \mathbf{r}_u\times\mathbf{r}_v = \langle\frac{\partial(y,z)}{\partial(u,v)}, \frac{\partial(z,x)}{\partial(u,v)}, \frac{\partial(x,y)}{\partial(u,v)}\rangle$, formula for normal vector to a surface given by the graph of a function $z = h(x,y) : \mathbf{N}(x,y) = \langle -h_x, -h_y, 1\rangle$, giving the formula

$$\int\int_S f(x,y,z)dS = \int\int_D f(x,y,h(x,y))\sqrt{1 + (h_x)^2 + (h_y)^2}\, dxdy,$$

formulas for surface integrals with respect to coordinate elements, surface integrals of the form $\int\int_S Pdydz + Qdzdx + Rdxdy$, surface integrals and vector fields $\mathbf{F} = \langle P, Q, R \rangle$,

$$\int\int_S \mathbf{F} \cdot \mathbf{n}\, dS \ = \ \int\int_S P dydz + Q dzdx + R\, dxdy,$$

$$\int\int_S \mathbf{F} \cdot \mathbf{n}\, dS \ = \ \int\int_D \mathbf{F}(\mathbf{r}(u,v)) \cdot \mathbf{N}(u,v)\, dA$$

if $\mathbf{r} : D \to S$ is a parametrization of the surface S, the flux of a vector field across a surface S : $\Phi = \int\int_S \mathbf{F} \cdot \mathbf{n}\, dS$.
Review hw (Assignments 33 and 34) and try problems pp. 1057- 1059, 2, 14, 15, 19.

(i) Section 14.6 : The Divergence Theorem: If S is a closed piecewise smooth surface bounding the space region T and $\mathbf{n}$ is the outer unit normal vector of S, then if $\mathbf{F}$ is continuously differentiable on T,

$$\int\int_S \mathbf{F} \cdot \mathbf{n}\, dS = \int\int\int_T \nabla \cdot \mathbf{F}\, dV,$$

examples and applications of the divergence theorem.
Review hw and try problems pp. 1065-1066, 1, 3, 8, 11, 13.

(j) Section 14.7 : Stokes' Theorem: If S is an oriented bounded piecewise smooth surface with positively oriented boundary $\mathbf{C}$ and unit normal $\mathbf{n}$, and if the vector field $\mathbf{F}$ is continuously differentiable in a space region containing S, then

$$\int_{\mathbf{C}} \mathbf{F} \cdot \mathbf{T}\, ds \;=\; \int\int_S \text{curl} \cdot \mathbf{n}\, dS,$$

examples and applications, irrotational vector fields in simpley connected regions, Theorem: If $\mathbf{F}$ is continuosly differentiable in a simply connected region D in space, then $\mathbf{F}$ is irrotational if and only if $\mathbf{F}$ is conservative; i.e. $\nabla \times \mathbf{F} \equiv \mathbf{0}$ in D if and only if $\mathbf{F} = \nabla\phi$ for some scalar function ϕ defined on D, calculating potential functions for irrotational vector fields defined in simply connected regions.
Review hw and try problems 4, 5, 9, 10, 11, 14.

2. Try some problems from the miscellaneous problems sections of the book,
pp. 1012 following, nos. 43, 44, 52.
pp. 1074 and following, nos. 1, 3, 4 8, 11, 13, 17, 19.