University of Colorado
Department of Mathematics

2005/2006 Semester 1   Math 2400 Calc 3   Discussion and practice, midterm 3

There is no need to try all the problems listed. I was just asked to list some problems from the book that might be helpful. The HW problems are the most helpful.

1. Material from book needed for exam:

   (a) Sections 9.2, 11.8, Polar, cylindrical and spherical coordinates: although not "officially" on this midterm, if you need to, you should briefly review this material as an aid with integration problems involving polar, cylindrical, or spherical coordinates.

   (b) Section 12.8, Directional derivatives and the gradient vector: directional derivative, definition and examples; calculation of the directional derivatives in terms of the dot product of the gradient and the directional unit vector; the gradient vector and its interpretation and significance; the gradient vector for F and its interpretation as a normal vector to the surface $F(x, y, z) = 0$.
   Review homework and try problems pp. 917-917, 3, 5, 17, 14, 23, 25, 31.

   (c) Section 12.9, Lagrange multipliers and constrained optimization: the method of Lagrange multipliers with one constraint, i.e. finding max or min of f subject to the one constraint $g = 0$, where f and g are both continuously differentiable functions of two variables; finding max or min of f subject to the one constraint $g = 0$, where f and g are both continuously differentiable functions of three variables; problems that have two constraints, i.e. finding the max or min of f subject to the two constraints $g = 0$ and $h = 0$ where $f, g,\ h$ are continuously differentiable.
   Review hw (Assignments 18 andn 19) and try problems pp. 926-927, 5, 7, 13, 17, 35.

   (d) Section 13.1, Double integrals: definition and examples of the double integral over a rectangle, partitions and Riemann sums over over rectangles, definition of the double integral of a continuous function over a rectangle $\mathcal{R}$ as a limit of Riemann sums, iterated integrals and using iterated integration to calculate double integrals over rectangles; independence of the value of the integral with respect to order of integration in iterated integrals over rectangles, iterated integrals and cross sections.
   Review hw and try problems pp. 947-949, 7, 17, 28, 31.

   (e) Section 13.2, Double integrals over more general regions: definition of double integral of a continuous function over a more general planar region as a limit of

Riemann sums; evaluation of double integrals over vertically simple and horizontally simple regions via iterated integrations (vertically simple - integrate with respect to y first, horizontally simple - integrate with respect to x first, changing the order in iterated integration over regions that are both vertically and horizontally simple; properties of double integrals.
Review hw and try problems pp. 955-956, 3, 7, 13, 17, 24, 29, 30.

(f) Section 13.3, Area and volume by double integration: definition of the volume of the solid below the graph $z = f(x, y)$ and above a planar region $\mathcal{R}$, calculating volume via the method of iterated integration, using the special case $z = 1$ to calculate via double integrals the area of a planar region $\mathcal{R}$, the volume bounded above and below two surfaces, calculating the volumes via iterated integration.
Review hw and try problems pp. 961-962, 3, 9, 15, 17, 23, 29.

(g) Section 13.4 : Double integrals in polar coordinates; polar rectangles, polar partitions of polar rectangles, definition of the double integral over a polar rectangle as limit of Riemann sums, $dA = rdrd\theta$ for $r \geq 0$, calculating double integrals over polar integrals via iterated integration, more general polar-coordinate regions, radially simple polar regions, calculating double integrals over radially simple regions via iterated integration (integrate with respect to r first), changing coordinates from Cartesian to polar, calculating area and volume using polar coordinates, $V = \int\int_{\mathcal{R}} (z_{\text{top}} - z_{\text{bottom}})dA$, evaluating improper integrals of the type $\int_0^\infty e^{-x^2}dx$ using polar coordinates,
Review hw and try problems pp. 968-970, 2, 3, 11, 15, 17, 27.

(h) Section 13.5 : Application of double integrals: density function over laminar regions, mass of a laminar region expressed as the double integral of the density function over the region, centroid of a planar laminar region $\mathcal{R}$, use of symmetry to simplify calcuation of centroid coordinates, volume and first theorem of Pappus, surface area and second theorem of Pappus.
Review hw and try problems pp. 977- 978, 3, 5, 19.

(i) Section 13.6 : Triple integrals; Riemann sums over rectangle blocks, Riemann sums with respect to inner partition of a spatial region T, evaluating triple integrals of continuous functions over rectangular blocks via iterated integration, density function of a spatial region, mass of a spatial region with respect to a density function, centroid of a spatial region, Volume of T is equal to $\int\int\int_T 1dV$, iterated triple integrals over z-simple regions, $\int\int\int_T fdV = \int\int_{\mathcal{R}}(\int_{z_1(x,y)}^{z_2(x,y)} fdz)dA$, similar discussion for integrating functions over x-simple or y-simple spatial regions, calculating the volume of spatial regions bounded by two surfaces.
Review hw and try problems pp. 987- 989, 3, 9, 13, 19, 41, 43.

(j) Section 13.7 : Integration in cylindrical and spherical coordinates; the general formula for triple integrals in cylindrical coordinates: $dV = r dz dr d\theta$, calculating volumes, mass, centroids using cylindrical coordinates, spherical coordinate integrals, centrally simple spherical regions, the general formula for triple integrals in spherical coordinates: $dV = \rho^2 \sin\phi d\rho d\phi d\theta$, calculating volume, mass, centroids using spherical coordinates.
Review hw and try problems 10, 11, 12, 15, 19.

2. Try some problems from the miscellaneous problems sections of the book,
pp. 938 and following, nos. 25, 39.
pp. 1012 and following, nos. 3, 7, 8, 10, 11, 12, 13, 19, 24, 25.