University of Colorado
Department of Mathematics

2005/2006 Semester 1 Math 2400 Calc 3 Discussion and practice, midterm 1

There is no need to try all problems listed. I was just asked to list some from the book that might be helpful. The HW problems are the most helpful.

1. Material from book needed for exam:
   (a) Section 11.1, Vectors in the plane: vectors, components of vectors, position vectors, length of vectors algebraic operations with vectors (addition, subtraction, scalar multiplication etc.), unit vectors, basic unit vectors $\mathbf{i}$ and $\mathbf{j}$.
   Review Assignment 1 and try problems pp. 779. 17, 21, 27, 29

   (b) Section 11.2, three-dimensional vectors: rectangular coordinate system, distance formula, eq. for spheres of radius r, vectors in space, position vectors, length of vectors, components of vectors, unit vectors, basic unit vectors $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$, dot product, rules of dot product (eq 9 p. 784), geometric interpretation of dot product, perpendicular vectors and the dot product, direction angles and projections, component of $\mathbf{a}$ along $\mathbf{b}$, work problems.
   Review Assignment 2 and try problems pp. 788 5, 22, 27, 44, 53.

   (c) Section 11.3, the cross product of vectors, definition and calculation of 2×2 determinants, definition and calculation of 3×3 determinants, definition and calculation of the vector cross product $\mathbf{a} \times \mathbf{b}$, geometrical significance of the cross product, $\mathbf{a} \times \mathbf{b}$ is perpendicular to $\mathbf{a}$ and $\mathbf{b}$, Two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel if and only if their crossed product is $\mathbf{0}$, area of parallelograms and triangles using cross products, algebraic properties of cross product and dot product, the scalar triple product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$, volumes of prisms and parallelepipeds using scalar triple product.
   Review Assignment 3 and try problems pp.796, 3, 8, 14, 21.

   (d) Section 11.4 Lines and planes in space: vector equation of a line in space, parametric equation of a line in space, paralletl lines, skew lines, planes in space, normal vector to a plane, determining the equation of a plane in the form
   $$ax + by + cz = d$$
   knowing one point on the plane and a normal vector to the plane, determining an equation for the plane knowing three non-collinear points on the plane, angle between intersecting planes, expressing a line as an intersection of planes.
   Review Assignment 4 and try problems pp. 803 3, 7, 9, 14, 27, 31, 34, 39.

   (e) Section 11.5 Curves and motion in space: parametric curves $\mathcal{C}$, coordinate functions of a parametric curve, vector-valued functions of a single real variable and their relationship to curves, the derivative of a vector-valued function,

component-wise differentiation of vector valued function, finding the equation of the tangent line to a curve through a point on that curve, rules of differentiation, position vectors and the corresponding velocity and acceleration vectors, speed and scalar acceleration as a function of t, integration of vector valued functions, recovering position vector from knowledge of acceleration function along with initial accelaration vector and initial velocity vector, projectile motion problems involving gravity and force, initial velocity and initial acceleration, and angles of inclination, angles of sight, etc.
Review Assignment 5 and relative part of Assignment 6, try problems pp. 816, 15, 24, 33, 52, 64.

(f) Section 11.6 Curvature and acceleration: arc length s of a smooth curve in space, arc-length parametrization for a curve, plane curves, Unit tangent vector $\mathbf{T}(t)$ for a plane curve, cuvations κ of a plane curve, various formulas for κ if $\mathbf{r}(t) =< x(t), y(t) >$ (eq. (12)) and if $\mathbf{r}(x) =< x, y(x) >$ (eq (13)) , unit normal vector $\mathbf{N}$ to a curve, short mention of radius of curvature $\rho = \frac{1}{\kappa}$, curvature of space curves, unit tangent vector $\mathbf{T}$, curvature κ of a space curve- Eq. (19) p. 823, unit normal vector $\mathbf{N}$ of a space curve, eq. (20), (21), tangential component of acceleration $a_T = \frac{dv}{dt}$ (eq. (23)), normal component of acceleration $a_N = \kappa v^2$ Eq (24), formula for acceleration $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$, solve for $\mathbf{N} = \frac{\mathbf{a}-a_T\mathbf{T}}{a_N}$ (eq (29)), note the different formulas for curvature κ in eq (27) and normal component of acceleration (28) that are often easier to compute than eq. (19). **Note material pp. 827 - 830, "Newton, Kepler, and the solar system" and following not covered in exam**.
Review relative part of Assignment 6 and Assignment 7, try problems pp. 830 1, 5, 9, 10, 13, 19, 25, 33, 43.

(g) Section 11.7 cylinders and quadric surfaces, graphs of equations $F(x, y, z) = 0$, planes and traces of surfaces, cylinders created from curves in a plane, right circular cylinders, other types of criculars, rulings of a cylinder (always paralletl to axis correponding to missign variable), surfaces of revolution constructed from revolving a curve in in a plane (usually the xy plane, yz plane, or xz plane) around a line in that plane (usually the line is a coordinate axis), how to construct the equation of a surface of revolution given the surface of the curve, quadric surfaces with equations of the form

$$Ax^2 + By^2 + Cz^2 + Dx + Ey + Fz + H = 0,$$

ellipsoids, elliptic parabaloid, elliptical cones, hyperboloid of one sheet, hyperboloid of two sheets, hyperbolic paraboloids, their traces, their equations.
Review Assignment 8, try problems pp. 839 1, 3, 7, 17, 20, 33, 35, 44.

2. Try some problems from the miscellaneous problems sections of the book, pp. 847 and following, nos. 7,8,11,16, 19, 21, 22, 23.