1. (20) Determine whether the following matrices are (a) in reduced row echelon form, (b) in row echelon form but not reduced row echelon form or (c) neither. If (b) applies, explain briefly why the matrix is not in reduced echelon form, and if (c) applies, explain briefly why the matrix is not in echelon form.

(i) $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$;

(ii) $\begin{bmatrix} 1 & -3 & 3 \\ 0 & 14 & -7 \\ 0 & 4 & -2 \end{bmatrix}$;

(iii) $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & -2 \end{bmatrix}$;

(iv) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

2. (20) Let A be the matrix

$$A = \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}.$$

(i) Reduce the matrix A to reduced row echelon form. (Points will be deducted for not using **elementary** row operations.)

(ii) Do the columns of A span $\mathbb{R}^3$? Why or why not?

(iii) Are the columns of A linearly independent? Why or why not?

3. (15) Let A be the 3×6 matrix of Question 2. Use your answer to that problem to solve the equation $A\mathbf{x} = \mathbf{0}$ in parametric vector form.

4. (15) Let

$$\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 16 \\ 18 \end{bmatrix},$$

and let A be the matrix of Question 2.

(i) Verify that $\mathbf{v}$ is a solution to the equation $A\mathbf{x} = \mathbf{b}$. [**Caution**: do not confuse the original matrix A with its reduced echelon form.]

(ii) Use this fact about $\mathbf{v}$ and the result of Question 3 to find the general solution to the equation $A\mathbf{x} = \mathbf{b}$ in parametric vector form.

5. (30)

(i) Show that the map $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$S(x_1, x_2) = (x_1^2,\ 3x_1 - x_2)$$

is not linear, by finding an **explicit counterexample**.

(ii) The matrix A of Question 2 corresponds (via $T(\mathbf{x}) = A\mathbf{x}$) to a linear transformation $T : \mathbb{R}^6 \to \mathbb{R}^3$ of the form

$$T(x_1, x_2, x_3, x_4, x_5, x_6) = (\dots,\ \dots,\ \dots).$$

Fill in the gaps to give an explicit description of T in terms of coordinates.

(iii) Is T injective ("one-to-one" in the book)? Why or why not?

(iv) Is T surjective ("onto" in the book)? Why or why not?

6. [For **20 bonus points** up to a maximum of 100.] Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\}$ be a linearly independent set of vectors in $\mathbb{R}^n$, and let $T : \mathbb{R}^n \to \mathbb{R}^m$ be an injective linear transformation. (In the terminology of the book, this means that T is "one-to-one".) Show that the vectors $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_p)\}$ are linearly independent in $\mathbb{R}^m$.

Name: ______________________________

University of Colorado

Mathematics 3135: First In-Class Exam

September 21, 2016

Problem	Points	Score
1	20	
2	20	
3	15	
4	15	
5	30	
Total	100	