Math 3001 Analysis 1
Homework Set 6

Spring 2017

Course Instructor: Dr. Markus Pflaum

Contact Info: Office: Math 255, Telephone: 2-7717, e-mail: `markus.pflaum@colorado.edu`.

Problem 1: Let $\sum_{n=0}^{\infty} v_n$ and $\sum_{n=0}^{\infty} w_n$ be two series of complex numbers. Define their so-called *Cauchy product* as the series $\sum_{n=0}^{\infty} z_n$ defined by

$$z_n := \sum_{k=0}^{n} v_k \, w_{n-k} \, .$$

Prove that if both $\sum_{n=0}^{\infty} v_n$ and $\sum_{n=0}^{\infty} w_n$ are absolutely convergent, then their Cauchy product $\sum_{n=0}^{\infty} z_n$ is absolutely convergent as well, and

$$\Big(\sum_{n=0}^{\infty} v_n \Big) \cdot \Big(\sum_{n=0}^{\infty} w_n \Big) = \sum_{n=0}^{\infty} z_n \, .$$

(5P)

Problem 2: Derive from Problem 1 that $e^{z+w} = e^z \cdot e^w$ for all $z, w \in \mathbb{C}$. (3P)

Problem 3: Derive from Problem 2 the addition theorems for the trigonometric functions cos and sin. (4P)

Problem 4: a) Determine for the following function whether it is continuous at the point $x_0 = 0$.

$$f : \mathbb{R} \to \mathbb{R}, \quad x \mapsto f(x) = \begin{cases} \frac{x-6}{x-3} & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ \sqrt{4 + x^2} & \text{if } x > 0 \end{cases}$$

(4P)

b) Define the maximal domain $D \subset \mathbb{R}$ on which

$$g : D \to \mathbb{R}, \quad x \mapsto g(x) = \frac{x^2 + 3x + 5}{x^2 + 3x - 4}$$

is a continuous function. (4P)