Math 3001 Analysis 1
Homework Set 5

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Problem 1: Use the comparison, root or ratio test to determine whether the following series converge or diverge:

$$\text{a)} \quad \sum_{n=0}^{\infty} \frac{2^n n^3}{3^n} \qquad \text{b)} \quad \sum_{n=0}^{\infty} \frac{5^n}{3^n(n^4+2)} \qquad \text{c)} \quad \sum_{n=1}^{\infty} \left(\frac{1}{5} + \frac{1}{n}\right)^n \qquad \text{d)} \quad \sum_{n=0}^{\infty} \frac{1}{n^n}$$

(8P)

Problem 2: Prove for $x, y > 0$ the following inequality:

$$|\sqrt{x} - \sqrt{y}| \leq \frac{|x-y|}{2\min\{\sqrt{x}, \sqrt{y}\}}.$$

Derive from this that the function $\mathbb{R}^+ \to \mathbb{R}$, $x \mapsto \sqrt{x}$ is continuous. (4P)

Problem 3: a) Show that for $x \in \mathbb{R}$ and all $n \in \mathbb{N}^*$

$$x^n - 1 = (x-1)\left(1 + x + \ldots + x^{n-1}\right)$$

(2P)

b) Let $n, m \in \mathbb{N}^*$, and consider the function

$$f : \mathbb{R} \setminus \{1, -1\} \to \mathbb{R}, \quad x \mapsto \frac{x^n - 1}{x^m - 1}.$$

Determine the limit $\lim_{x \to 1} f(x)$. (2P)

Problem 4: For $k \in \mathbb{N}^*$ determine the limits

$$\lim_{t \to 0} \frac{1}{t^k} \exp\left(-\frac{1}{t^2}\right)$$

(4P)