Math 3001 Analysis 1
Homework Set 4

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Problem 1: Show that the set $\mathbb{C} := \mathbb{R}^2$ together with addition

$$+ : \mathbb{C} \times \mathbb{C} \to \mathbb{C},\ \big((a,b),(c,d)\big) \mapsto (a+c, b+d)\,,$$

multiplication

$$\cdot : \mathbb{C} \times \mathbb{C} \to \mathbb{C}, \big((a,b),(c,d)\big) \mapsto (ac-bd, bc+ad)\,,$$

zero element $(0,0)$, and one element $(1,0)$ is a field. It is called the *field of complex numbers.* (6P)

Problem 2: Prove that the field $\mathbb{C}$ together with the absolute value as norm is complete. Recall: The norm of $z = (a,b) = a + ib$ is defined by $|z| := \sqrt{a^2+b^2}$. (4P)

Problem 3: Prove by induction the following formulas:

$$\text{a)}\quad \sum_{k=1}^{2n}(-1)^{k+1}\frac{1}{k} = \sum_{k=1}^{n}\frac{1}{n+k}, \qquad \text{b)}\quad \prod_{k=2}^{n}\Big(1-\frac{1}{k^2}\Big) = \frac{n+1}{2n}.$$

(6P)

Problem 4: Determine the following limits:

$$\text{a)}\quad \lim_{x\to -3}\frac{3x+9}{x^2-9} \qquad \text{b)}\quad \lim_{x\to 3}\frac{x^3-5x+4}{x^2-2}\,.$$

(4P)