Math 3001 Analysis 1
Homework Set 3

Spring 2017

Course Instructor: Dr. Markus Pflaum

Contact Info: Office: Math 255, Telephone: 2-7717, e-mail: `markus.pflaum@colorado.edu`.

Problem 1: Show that the sequences $\left(\frac{n}{n+2}\right)_{n\in\mathbb{N}}$ and $\left(\frac{n}{2^n}\right)_{n\in\mathbb{N}}$ converge and determine their limits. (4P)

Problem 2: Prove that for every natural number $n \neq 0$:

a)
$$\left(1+\frac{1}{n}\right)^n \leq \sum_{k=0}^{n} \frac{1}{k!} < 3,$$

b)
$$\left(\frac{n}{3}\right)^n \leq \frac{1}{3}\, n!\,.$$

(6P)

Problem 3: Prove that for real $a, b \geq 0$ one has

$$\sqrt{a\,b} \leq \frac{1}{2}(a+b).$$

(2P)

Problem 4: Show that for each natural $k \geq 2$ the series $\sum\limits_{n=1}^{\infty} \frac{1}{n^k}$ converges.

Hint: You can use the result proven in class that the series $\sum\limits_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges. (4P)

Problem 5: Check which of the following sequences converges, and determine its limit, if it does:

a) $x_n = \frac{1+(-1)^n}{n}$ b) $x_n = \frac{n^3}{n^2+1}$

c) $x_n = \frac{n^2}{n^2+1}$ d) $x_n = \frac{n^2-n}{n^3+1}$

(4P)