Math 3001 Analysis 1
Homework Set 1

Spring 2017

Course Instructor: Dr. Markus Pflaum

Contact Info: Office: Math 255, Telephone: 2-7717, e-mail: `markus.pflaum@colorado.edu`.

Problem 1: Let M, N, L be sets.
a) Prove the following rule of de Morgan:

$$M \setminus (N \cup L) = (M \setminus N) \cap (M \setminus L).$$

b) Prove the following distributivity law:

$$M \cap (N \cup L) = (M \cap N) \cup (M \cap L).$$

(4P)

Problem 2: Let $f : X \to Y$ and $g : Y \to Z$ be mappings. Prove the following claims:

a) If f and g are injective, then $g \circ f$ is injective as well.

b) If f and g are surjective, then $g \circ f$ is surjective, too.

(4P)

Problem 3:

a) Let $f : X \to Y$ be a mapping, and $A, B \subset Y$. Show that then

$$f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$$
$$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B).$$

b) Determine, whether the following equalities are true for subsets $C, D \subset X$:

$$f(C \cap D) = f(C) \cap f(D)$$
$$f(C \cup D) = f(C) \cup f(D).$$

(6P)

Problem 4: Prove the following statements for all positive integers n and real numbers $q \neq 1$:

a) $\sum\limits_{k=1}^{n} k^3 = \left(\sum\limits_{k=1}^{n} k \right)^2 = \frac{n^2(n+1)^2}{4}$,

b) $\sum\limits_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q}$ (finite geometric series).

(6P)