Math 2002 Number Systems
Homework Set 4

Fall 2022

Course Instructor: Dr. Markus Pflaum

Contact Info: Office: Math 255, Telephone: 2-7717, e-mail: `markus.pflaum@colorado.edu`.

Problem 1: Let $f : X \to Y$ be a function for which there exist functions $g_1 : Y \to X$ and $g_2 : Y \to X$ such that $g_1 \circ f = \mathrm{id}_X$ and $f \circ g_2 = \mathrm{id}_Y$. Show that then f is invertible and that $g_1 = g_2$. (4P)

Problem 3:

a) Let $f : X \to Y$ be a mapping, and $A, B \subset Y$. Show that then

$$f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$$
$$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B).$$

b) Determine, whether the following equalities are true for subsets $C, D \subset X$:

$$f(C \cap D) = f(C) \cap f(D)$$
$$f(C \cup D) = f(C) \cup f(D).$$

(4P)

Problem 4: Show that for all $x, y \in \mathbb{R}$

$$\max\{x, y\} = \frac{1}{2}(x + y + |x - y|) \quad \text{and} \quad \min\{x, y\} = \frac{1}{2}(x + y - |x - y|)$$

(4P)

Problem 5: Consider the triple $F = (\mathbb{R}, \mathbb{R}, \Gamma)$ with

a) $\Gamma = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x^2 + y^2 = 1\}$,

b) $\Gamma = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid x = y^2 + 1\}$,

c) $\Gamma = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid y = x^2 + 1\}$.

d) $\Gamma = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid \sin y = \cos x\}$.

In which of these cases is F a function? Explain! (4P)

Problem 6:

a) Let $n \in \mathbb{N}_{>0}$. Find and prove by induction a formula for $\sum_{k=1}^{n} \frac{1}{k(k+1)}$. (2P)

b) Prove by induction the following formula for positive natural n:

$$\prod_{k=2}^{n} \left(1 - \frac{1}{k^2}\right) = \frac{n+1}{2n} .$$

(2P)