De Morgan's Laws

Intro to Analysis

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Set-up and notation

Here our underlying set is $\mathbb{R}$.

Definition

Let $S \subset \mathbb{R}$. The complement of S in $\mathbb{R}$, written $\mathbb{R}\backslash S$, is defined by

$$\mathbb{R}\backslash S = \{x \in \mathbb{R} : x \notin S\}.$$

Example

If $S = [0, 1]$, then

$$\mathbb{R}\backslash S = (-\infty, 0) \cup (1, \infty).$$

Augustus De Morgan

Here is a photograph of Augustus De Morgan, mathematician and logician (1806–1871):

De Morgan's Laws: how to take complements of unions and intersections

Theorem (De Morgan's Laws)

Let A and B be subsets of $\mathbb{R}$.

$$\mathbb{R}\backslash(A \cup B) = (\mathbb{R}\backslash A) \cap (\mathbb{R}\backslash B)$$

and

$$\mathbb{R}\backslash(A \cap B) = (\mathbb{R}\backslash A) \cup (\mathbb{R}\backslash B).$$

Example

Let $A = (2, 4]$, and $B = [3, 5)$. Then $A \cap B = [3, 4]$ and

$$(\mathbb{R}\backslash A) \cup (\mathbb{R}\backslash B) = \mathbb{R}\backslash(A \cap B) = \mathbb{R}\backslash[3, 4]$$

$$= (-\infty, 3) \cup (4, \infty).$$