University of Colorado
Department of Mathematics

2019/2020 Semester 2 | Math 6320 Real Analysis 2 | Assignment 4

Due Friday March 20, 2020 (email me scanned copies of your solutions)

1. Let $(X, \mathcal{M}, \mu)$ be a measure space, and suppose that $p, q,\ r\ \in\ (1, \infty)$ satisfy
$$\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1.$$
If $f\ \in\ L^p(\mu),\ g\ \in\ L^q(\mu)$, and $h\ \in\ L^r(\mu)$, prove that $f \cdot g \cdot h\ \in\ L^1(\mu)$ and that
$$\|f \cdot g \cdot h\|_1\ \leq\ \|f\|_p \cdot \|g\|_q \cdot \|h\|_r.$$

2. Let $(X, \mathcal{M}, \mu)$ be a measure space.
   (a) Fix $p \in [1, \infty)$. If $f \in L^p(X, \mu) \cap L^\infty(X, \mu)$, prove that $f \in L^r(X, \mu)$ for all $r \in (p, \infty)$.
   (b) Fix $p \in [1, \infty)$. If $f \in L^p(X, \mu) \cap L^\infty(X, \mu)$, prove that
$$\|f\|_\infty\ =\ \lim_{r \to \infty} \|f\|_r.$$

3. Do the following problems in the Royden–Fitzpatrick textbook: p. 386, # 54 all parts, p. 399 #5.