

University of Colorado
Department of Mathematics

2019/2020 Semester 2

Math 6320 Real Analysis 2

Assignment 3

Due Wednesday March 11, 2020

1. Let (X, \mathcal{M}, μ) be a measure space. Let $f : X \rightarrow [-\infty, \infty]$ be a measurable function.
 - (a) Prove that if f is integrable over X with respect to μ , then $\{x \in X : f(x) \neq 0\}$ is of σ -finite measure.
 - (b) Prove that if f is integrable over X with respect to μ , then for every $\epsilon > 0$ there is a simple function ϕ such that

$$\int_X |f(x) - \phi(x)| d\mu < \epsilon.$$

2. Let μ and ν be two measures on the measurable space (X, \mathcal{M}) . Prove or disprove the following statements:
 - (a) $\mu \ll \nu$ and $\mu \perp \nu \Rightarrow \mu \equiv 0$.
 - (b) $\mu \ll \nu$ and $\nu \ll \mu \Rightarrow \mu = \nu$.
3. Do the following problems in the Royden–Fitzpatrick textbook: p. 346: 14, 15; p. 352: 18; p. 365: 12, 15; p. 380: 45.