University of Colorado
Department of Mathematics

Due Friday Feb. 14, 2020

1. Suppose that f is in $C[0, 1]$, and

$$\int_0^1 f(x)x^n \, dx \; = \; 0 \; (n = 0, \; 1, \; 2, \; \cdots).$$

   Prove that $f(x) \; = \; 0 \; \forall x \in \; [0, 1]$.
   [Hint: Consider $\int_0^1 (f(x))^2 \, dx$ and use the Weierstrass Approximation Theorem.]

2. Let ν be a signed measure on the measurable space $(X, \mathcal{B})$. Let (A, B) and (A', B') be two Hahn decompositions for ν. Prove that $\nu(E \cap A) \; = \; \nu(E \cap A')$ and $\nu(E \cap B) \; = \; \nu(E \cap B')$, for every E in $\mathcal{B}$.

3. Do the following problems in the Royden– Fitzpatrick textbook: pp. 251–2, #27, 29, 34; pp. 341-342 #3, 4, 7.