University of Colorado
Department of Mathematics

2018/2019 Semester 2 — MATH 4330 — Second Midterm Exam, Takehome

Due Wednesday April 10, 2019, 5 p.m. in MATH 227.

No late papers will be accepted. You will **not** receive extra credit for doing parts of the 5330 take-home exam.

INSTRUCTIONS: You are to work by yourself. You are allowed to use the textbook, class notes, previous homework assignments, the previous exam, and any other book you find helpful; please cite your references. If you need any clarification about a problem, you should consult me, and not other students.

1. Give complete solutions to problems 2.7.2 (pp. 120–121), 2.9.4 (p. 131; you may use the result from 2.9.3), 3.2.9 (p. 175), 3.7.7 (a), (b), (c) (p. 202), 5.1.5 (p. 264), 5.7.4 (p. 295) and 6.2.2 (p. 311) of the Stade textbook.

2. Using the discussion in Proposition 3.7.1 p. 195 as a model, prove that if the piecewise continuous functions f, $g \in L^2[-\pi, \pi]$ have trigonometric Fourier coefficients $\{a_n\}_{n \geq 0} \cup \{b_n\}_{n \geq 1}$ and $\{\alpha_n\}_{n \geq 0} \cup \{\beta_n\}_{n \geq 1}$, respectively, then

$$\frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\overline{g(x)}dx \;=\; \frac{a_0\overline{\alpha_0}}{2} + \sum_{n=1}^{\infty}[a_n\overline{\alpha_n} + b_n\overline{\beta_n}].$$

3. Find the values of the real constants B_1, B_2, B_3 such that

$$\int_0^2 |\frac{3}{4} - (B_1 \sin{(\frac{\pi x}{2})} + B_2 \sin{(\pi x)} + B_3 \sin{(\frac{3\pi x}{2})})|^2 dx$$

achieves its minimum possible value. Be sure to explain your reasoning.