University of Colorado
Department of Mathematics

2018/19 Semester 2 | Math 6320 Real Analysis 2 | Assignment 5

**Due Wednesday April 24, 2019**

1. Do the following problems in the Folland textbook: p. 188 $\#1, 9, 13, 14$; p. 197, #32; p. 199, #38.

2. (Past-year prelim problem!!) The *Fourier transform*, denoted by $\hat{f}$, of a function $f : \mathbb{R} \to \mathbb{R}$, is defined by

$$\hat{f}(s) \ = \ \int_{\mathbb{R}} f(x)\, e^{-2\pi i s x} dx,$$

for any $s \in \mathbb{R}$ such that the integral on the right exists in the sense of Lebesgue.

(a) Show that if $f \in L^1(\mathbb{R})$, then $\hat{f}(s)$ exists for every $s \in \mathbb{R}$, and $\hat{f}$ is bounded and continuous on $\mathbb{R}$, and

$$\sup_{s\in\mathbb{R}} |\hat{f}(s)| \ \leq \ \|f\|_{L^1(\mathbb{R})}.$$

(b) Show that if $f,\ g \in L^1(\mathbb{R})$, then

$$\int_{\mathbb{R}} \hat{f}(u) g(u) du \ = \ \int_{\mathbb{R}} f(v) \hat{g}(v) dv.$$