University of Colorado
Department of Mathematics

2018/19 Semester 2 | Math 6320 Real Analysis 2 | Assignment 4

Due Friday March 22, 2019

1. Do the following problems in the Folland textbook: p. 177–178, #55(a), 56, 59, 60, 61, 62(a), (b), (c).

2. (Past-year prelim problem!!)
   (a) If $\{f_1, f_2, \cdots, f_N, \cdots, \}$ is a complete orthonormal set in the Hilbert space $L^2[0,1]$, where $[0,1]$ is equipped with Lebesgue measure, and B is an arbitrary measurable subset of positive measure in $[0,1]$, prove that

$$1 \leq \int_B \sum_{i=1}^{\infty} |f_i(x)|^2 dx.$$

   (b) Deduce that $\sum_{i=1}^{\infty} |f_i(x)|^2 = \infty$ a.e.