University of Colorado Department of Mathematics

2018/2019 Semester 2

Math 6320 Real Analysis 2

Assignment 3

Due Wednesday, March 6, 2019

1. (a) Show that the space ${\bf c}$ of all convergent sequences of complex numbers is a Banach space, where we define

$$\{c_n\}_{n=1}^{\infty} + \{d_n\}_{n=1}^{\infty} = \{c_n + d_n\}_{n=1}^{\infty} \in \mathbf{c},$$

$$\alpha \cdot \{c_n\}_{n=1}^{\infty} \text{ for } = \{\alpha \cdot c_n\}_{n=1}^{\infty} \in \mathbf{c}, \text{ for } \alpha \in \mathbb{C},$$

$$\|\{c_n\}_{n=1}^{\infty}\| = \sup_{n \in \mathbb{N}} \{|c_n\|\}.$$

- (b) Show that the subspace \mathbf{c}_0 of \mathbf{c} , consisting of all complex sequences that converge to 0 is also a Banach space (the norm begin the same as the norm defined for \mathbf{c} in part (a) above.)
- 2. Do the following problems in the Folland textbook: p. 142 #68, pp. 154–156: # 2, 4, 7 (a), (b), 12 (a), (b), (c), (d); p. 164 # 27, 29 (a), (b).