University of Colorado
Department of Mathematics

2018/2019 Semester 2 | Math 6320 Real Analysis 2 | Assignment 2

Due Monday, February 11, 2019

1. Prove the reverse direction in Folland p. 109 # 42 (a), i.e. prove that if the chordal slope inequality is satisfied for $\phi : (a,b) \to \mathbb{R}$ for all $a < s \leq s' < t' < b$ and $a < s < t \leq t' < b$, then ϕ is convex.

2. (a) If $\{\lambda_i\}_{i=1}^{n}$ is a finite sequence of positive numbers whose sum is 1, and if $\phi : (a,b) \to \mathbb{R}$ is convex, and if $a < x_1 \leq x_2 \leq \cdots \leq x_i \leq \cdots \leq x_n < b$, prove that
$$\phi(\sum_{i=1}^{n} \lambda_i \cdot x_i) \leq \sum_{i=1}^{n} \lambda_i \phi(x_i).$$

   (b) If $\{\lambda_i\}_{i=1}^{\infty}$ is an infinite sequence of positive numbers whose sum is 1, and if $\phi : (a,b) \to \mathbb{R}$ is convex, and if $\{x_i\}_{i=1}^{\infty}$ is a sequence satisfying $a < x_1 \leq x_2 \leq \cdots \leq x_i \leq \cdots \leq x_n \leq x_{n+1} < \cdots < b$, prove that
$$\phi(\sum_{i=1}^{\infty} \lambda_i \cdot x_i) \leq \sum_{i=1}^{\infty} \lambda_i \phi(x_i).$$

3. Let $\{\alpha_n\}_{n=1}^{\infty}$ be a sequence of positive numbers whose sum is 1, and let $\{\zeta_n\}_{n=1}^{\infty}$ be a sequence of positive numbers. Prove that
$$\prod_{n=1}^{\infty} [\zeta_n]^{\alpha_n} < \sum_{n=1}^{\infty} \alpha_n \zeta_n.$$

   [Hint: use convexity in some way.]

4. Do the following problems in the Folland textbook: p. 138: #59, # 63, # 65.