University of Colorado
Department of Mathematics

2018/2019 Semester 2 | Math 6320 Real Analysis 2 | Assignment 1

Due Monday, January 28, 2019

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a function of bounded variation on $\mathbb{R}$ so that f is of bounded variation on $[a, b]$ for every closed and bounded interval $[a, b] \subset \mathbb{R}$. Prove that the set of discontinuities of f is countable in cardinality, and that f' exists almost everywhere on $\mathbb{R}$. You may assume that f can be expressed as the difference of two monotone increasing functions on $\mathbb{R}$.
2. Let F, G be real-valued functions in BV and let $c \in \mathbb{R}$. Prove that $F + G \in BV$ and $c \cdot F \in BV$ and that
$$T_{F+G}(x) \leq T_F(x) + T_G(x) \ \forall x \in \mathbb{R}, \text{ and } \ T_{cF}(x) = |c| T_F(x) \ \forall x \in \mathbb{R}.$$
3. Let $F(x) = x^2 \sin\left(\frac{1}{x^2}\right)$ for $0 < x \leq 1$ and $F(0) = 0$. Prove that F is not of bounded variation on $[0, 1]$.
4. Let $F : [a, b] \to \mathbb{R}$. Suppose that $F'(x)$ exists for all $x \in [a, b]$ and F' is bounded on $[a, b]$. Prove that F is of bounded variation on $[a, b]$. Under the same hypotheses, prove that F is absolutely continuous on $[a, b]$.
5. Do the following problems in the Folland textbook: p. 107–108 # 29, 30. 35, 36 (a) and (b).