
 Workshop 4

1. Let $f(x) = \sqrt{9 - x^2}$, let $g(x) = \frac{2}{x}$ and let $h(x) = 5x^{2/3}$.

(a) What is the function $(f \circ g \circ h)(x)$, and what is its domain?

$$\begin{aligned}
 (f \circ g \circ h)(x) &= f(g(h(x))) \\
 &= f(g(5x^{2/3})) \\
 &= f\left(\frac{2}{5x^{2/3}}\right) \\
 &= \sqrt{9 - \left(\frac{2}{5x^{2/3}}\right)^2} \\
 &= \sqrt{9 - \frac{4}{25x^{4/3}}} \quad \text{or, simplified} \quad \frac{\sqrt{225x^{4/3} - 4}}{5x^{2/3}}
 \end{aligned}$$

Its domain is $\left(-\infty, -\left(\frac{2}{15}\right)^{3/2}\right] \cup \left[\left(\frac{2}{15}\right)^{3/2}, \infty\right)$.

(b) Find $(fgh)(x)$ and its domain.

$$(fgh)(x) = \sqrt{9 - x^2} \cdot \frac{2}{x} \cdot 5x^{2/3} = \frac{10\sqrt{9 - x^2}}{x^{1/3}}$$

Its domain is $[-3, 0) \cup (0, 3]$.

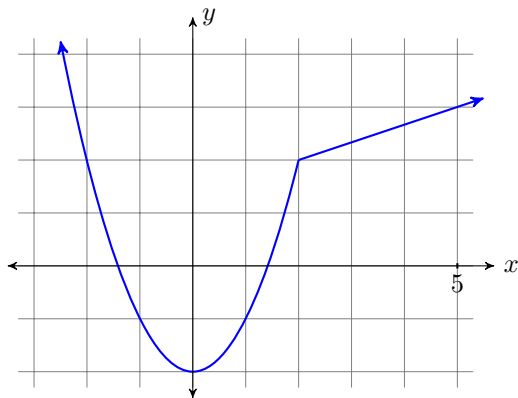
(c) Find $\left(\frac{fg}{h}\right)(x)$ and its domain.

$$\left(\frac{fg}{h}\right)(x) = \frac{\sqrt{9 - x^2} \cdot \frac{2}{x}}{5x^{2/3}} = \frac{2\sqrt{9 - x^2}}{5x^{5/3}}$$

Its domain is $[-3, 0) \cup (0, 3]$.

2. Graph the function $f(x) = \begin{cases} x^2 - 2, & \text{if } x \leq 2 \\ \frac{1}{3}x + \frac{4}{3}, & \text{if } x > 2 \end{cases}$.

Solution:



3. Graph the quadratic function $f(x) = -x^2 + 4x - 1$.

Solution:

