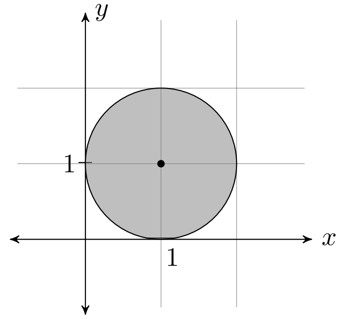
NAME ____________________________

Workshop 3

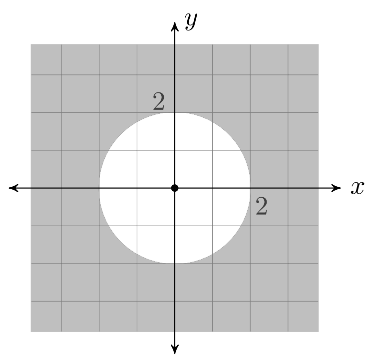
1. Graph the set of points in the x-y plane whose distance from the point $(1,1)$ is less than or equal to 1.

   The graph is the shaded area, the disk of radius 1 centered at (1,1), including the boundary circle:



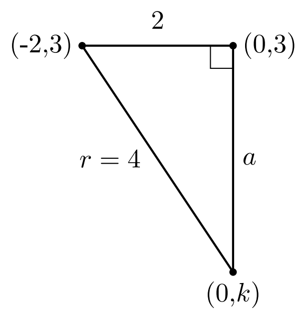
2. Graph the set $\{(x,y) \mid x^2+y^2>4\}$.

   The graph is the shaded area, everything outside the disk of radius 2 centered at (0,0), *not* including the boundary circle:



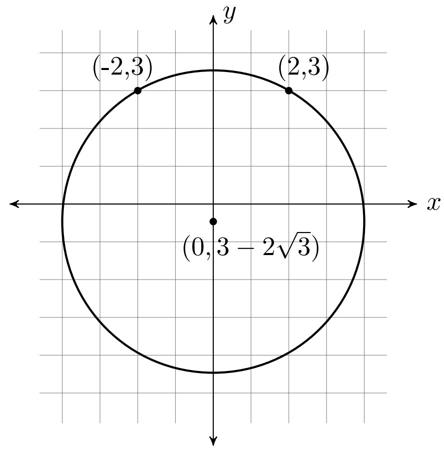
3. Find the equation of the circle passing through the points $(2,3)$ and $(-2,3)$ with radius 4, whose center lies below those points. Then graph the circle.

   Let's start with the information given: the radius, r, is 4, and the circle passes through the points $(2,3)$ and $(-2,3)$. There are two possible circles, one with center above and one with center below. We're told to look at the one with center below. Since the two points are at the same height, the x-coordinate of the circle's center must be in between them, by symmetry, and hence must be precisely 0. That is, if $C=(h,k)$ is the center, then $h=0$. So if we can find k we're done, because the equation is $(x-h)^2+(y-k)^2=r^2$, and we know h and r. How do we find k? The Pythagorean Theorem: we have a triangle,



By the Pythagorean Theorem, $a^2 + 2^2 = 4^2$, or $a = \sqrt{16-4} = \sqrt{12} = 2\sqrt{3}$. Thus, k is obtained from 3 by subtracting $2\sqrt{3}$, because we have to go down from $y = 3$ by $2\sqrt{3}$ to get to k.

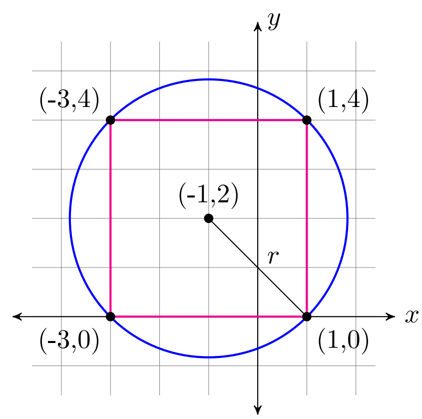
The picture is this:



And consequently the equation for this circle is $\boxed{x^2 + (y - 3 + 2\sqrt{3})^2 = 16}$.

4. Consider the square with sides of length 4 centered at $(-1, 2)$. Find the equation of the circle passing through the four vertices of the square.

   The picture is this:



   We are given the center of the circle, since it's the same as the center of the square, $C = (-1, 2)$. Thus, all we need to find it the radius. But this follows easily from the distance formula. It's the distance from the center $(-1, 2)$ to one of the four corners of the square, since the corners lie on the circle. Let's pick the bottom right hand corner, $(1, 0)$:

$$r = d\big((-1,2),(1,0)\big) = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

Thus, the equation, $(x-h)^2 + (y-k)^2 = r^2$, is

$$\boxed{(x+1)^2 + (y-2)^2 = 8}$$

5. Find the equation of the line passing through the point $(9, -3)$ and parallel to the line whose equation is $5x - 3y = 2$.

   We have a point $(9, -3)$, so all we need to find is the slope and the y-intercept. The slope is easily gotten from the line given, $5x - 3y = 2$, just solve for y, and it'll be the coefficient of x: Subtracting $5x$ from both sides gives

$$-3y = -5x + 2$$

   and dividing by -3 shows

$$y = \frac{5}{3}x - \frac{2}{3}$$

   Since the two lines are parallel, they have the same slope $\frac{5}{3}$, so our equation is

$$y = \frac{5}{3}x + b$$

   To get b, plug in the point given, with $y = -3$ and $x = 9$, then solve for b:

$$-3 = \frac{5}{3} \cdot 9 + b$$

   so $-3 = 15 + b$, and so $b = -18$. And we're done:

$$\boxed{y = \frac{5}{3}x - 18}$$