NAME _____

Workshop 2

1. Factor $121h^2 - 49$.

$$\boxed{(11h - 7)(11h + 7)}$$

2. Factor $18n^3 - 84n^2 + 98n$.

$$\boxed{2n(3n - 7)^2}$$

3. Factor $a^8 - 2a^4b^4 + b^8$. [Hint: use the formulas you know, for example the square of a sum. You should get 6 factors.]

$$\boxed{\begin{aligned} a^8 - 2a^4b^4 + b^8 &= (a^4)^2 - 2a^4b^4 + (b^4)^2 \\ &= \left[a^4 - b^4\right]^2 && \text{square of a sum} \\ &= \left[(a^2)^2 - (b^2)^2\right]^2 \\ &= \left[(a^2 - b^2)(a^2 + b^2)\right]^2 && \text{difference of squares} \\ &= \left[(a - b)(a + b)(a^2 + b^2)\right]^2 && \text{another difference of squares} \\ &= (a - b)^2(a + b)^2(a^2 + b^2)^2 && \text{distribute the square power} \end{aligned}}$$

4. Recall that if a and b are real numbers, then

$$\begin{aligned} a^2 - b^2 &= (a - b)(a + b) \\ a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\ a^4 - b^4 &= (a - b)(a^3 + a^2b + ab^2 + b^3) \\ &\vdots \\ a^n - b^n &= (a - b)(a^{n-1} + a^{n-2}b + \cdots + ab^{n-2} + b^{n-1}) \end{aligned} \tag{0.1}$$

(This just follows from distributing the bigger sum on the right-hand-side through $a-b$, then distributing a and $-b$ inside *it*, then canceling.) This also means that

$$\frac{a^n - b^n}{a-b} = a^{n-1} + a^{n-2}b + \cdots + ab^{n-2} + b^{n-1} \tag{0.2}$$

(just divide both sides of (0.1) by $(a-b)$), which is useful in calculating sums of the form $a^{n-1} + a^{n-2}b + \cdots + ab^{n-2} + b^{n-1}$. Use this fact (0.2) to calculate the sum

$$5 \cdot 32 + 5 \cdot 16 \cdot 3 + 5 \cdot 8 \cdot 9 + 5 \cdot 4 \cdot 27 + 5 \cdot 2 \cdot 81 + 5 \cdot 243$$

Show your work. [Hint: factor any common factor out and use (0.2).]

$$\begin{aligned}
5 \cdot 32 + 5 \cdot 16 \cdot 3 + 5 \cdot 8 \cdot 9 + 5 \cdot 4 \cdot 27 + 5 \cdot 2 \cdot 81 + 5 \cdot 243 & \\
&= 5(32 + 16 \cdot 3 + 8 \cdot 9 + 4 \cdot 27 + 2 \cdot 81 + 243) && \text{factor out the 5} \\
&= 5(2^5 + 2^4 \cdot 3 + 2^3 \cdot 3^2 + 2^2 \cdot 3^3 + 2 \cdot 3^4 + 3^5) && \text{rewrite as powers of 2 and 3} \\
&= 5 \cdot \frac{2^6 - 3^6}{2-3} && \text{use equation (0.2)} \\
&= 5 \cdot \frac{64 - 729}{-1} && \text{simplify} \\
&= 5 \cdot 665 \\
&= 3325
\end{aligned}$$

5. How many pounds of walnuts at \$0.85/lb should be mixed with 20lb of pecans at \$1.20/lb to give a mixture worth \$1.04/lb?

Let x denote the number of pounds of walnuts mixed. Then, we're mixing x pounds of walnuts with 20 pounds of pecans to get a total of $x + 20$ pounds of mixture. How much are we paying? Well, we're paying $\$0.85x$ for the walnuts plus $\$1.20 \cdot 20$ for the pecans. But this number better equal what we're paying for the mixture, $\$1.04 \cdot (x + 20)$. Thus, our equation is

$$0.85x + 1.20 \cdot 20 = 1.04(x + 20)$$

Distribute on the right side, and simplify $1.20 \cdot 20$,

$$0.85x + 24 = 1.04x + 20.8$$

then solve for x:

$$3.2 = 0.19x \implies 16.84 \approx x$$

so we're mixing approximately $\boxed{16.84 \text{ lbs}}$ of walnuts.