NAME ____________________

Workshop 1

1. Which of the following containments is true? Circle those which are correct.

$\boxed{\mathbb{N} \subseteq \mathbb{W}}$	$\mathbb{N} \supseteq \mathbb{W}$	$\boxed{\mathbb{W} \subseteq \mathbb{Q}}$	$\mathbb{Q} \supseteq \mathbb{H}$
$\mathbb{Z} \subseteq \mathbb{W}$	$\mathbb{Z} \supseteq \mathbb{Q}$	$\boxed{\mathbb{Z} \subseteq \mathbb{R}}$	$\boxed{\mathbb{H} \subseteq \mathbb{R}}$

2. State the containment relations between the reals $\mathbb{R}$, naturals $\mathbb{N}$, integers $\mathbb{Z}$, wholes $\mathbb{W}$ and rationals $\mathbb{Q}$. They will be of the form $A \subseteq B \subseteq C \subseteq D \subseteq E$. Are they proper containments?

$$\boxed{\mathbb{N} \subseteq \mathbb{W} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}, \qquad \text{Yes, proper.}}$$

3. The commutative property states that $a + b = b + a$ and $ab = ba$ for all real numbers a and b. What is the statement of the associative property?

$$\boxed{\begin{array}{c} a + (b + c) = (a + b) + c \\ a(bc) = (ab)c \end{array}}$$

4. What is the statement of the distributive property?

$$\boxed{a(b + c) = ab + ac}$$

5. We know that if a and b are real numbers such that $ab = 0$, then at least one of a and b has to be 0. By extension, given any real numbers $a_1, a_2, \dots, a_n$, if $a_1 a_2 \cdots a_n = 0$, then at least

one of them (perhaps several) must be 0. Now, the (rather ugly looking) polynomial

$$p(x) = \frac{\sqrt{6}}{10}\pi + \left(\frac{2\sqrt{6} - (-5\sqrt{2} + \sqrt{3} + \sqrt{6})\pi}{10}\right) x + \left(\frac{-2(-5\sqrt{2} + \sqrt{3} + \sqrt{6}) + (-5 - 5\sqrt{2} + \sqrt{3})\pi}{10}\right) x^2 + \left(\frac{-10(1 + \sqrt{2}) + 2\sqrt{3} + 5\pi}{10}\right) x^3 + x^4 \quad (0.1)$$

can be factored as

$$p(x) = (x - 1)\left(x - \sqrt{2}\right)\left(\frac{x + \sqrt{3}}{5}\right)\left(x + \frac{\pi}{2}\right) \quad (0.2)$$

A **root** of $p(x)$ is a real number a such that $p(a) = 0$, that is, replacing x with a in the polynomial will make the expression equal to 0. What are the roots of $p(x)$? (Hint: let $p(x) = 0$ and apply the fact noted above to $p(x)$ in its factored form (0.2).)

Suppose a is a real root of $p(x)$. Then,

$$\begin{aligned} p(a) &= (a - 1)\left(a - \sqrt{2}\right)\left(\frac{a + \sqrt{3}}{5}\right)\left(a + \frac{\pi}{2}\right) \\ &= 0 \end{aligned}$$

This means that at least one of $a - 1$, $a - \sqrt{2}$, $\frac{a+\sqrt{3}}{5}$ or $a + \frac{\pi}{2}$ must equal 0. These are the only possibilities, and they completely account for the four roots of $p(x)$, if we can extract a from each. Solving each of these for a gives

$a = 1$ $\qquad a = \sqrt{2}$ $\qquad a = -\sqrt{3}$ $\qquad a = -\frac{\pi}{2}$

6. Circle the numbers which are rational:

[2.12]	**[2.121212...]**	2.121221222...	$\sqrt{2}$
$\frac{\sqrt{2}}{2}$	$\frac{1}{3} + 2\pi$	$\frac{\sqrt{24}}{\sqrt{56}}$	**[$\frac{\pi^0}{237}$]**

7. Simplify the expression $\dfrac{27 - \sqrt{72}}{6}$.

$$\boxed{\frac{9 - 2\sqrt{2}}{2}}$$

8. Simplify the expression $-\left(\frac{27p^6}{8q^3}\right)^{\frac{2}{3}}$.

$$\boxed{-\frac{9p^4}{4q^2}}$$

9. Simplify the expression $(7\sqrt{2})^2$.

$$\boxed{98}$$

10. Simplify the expression $(3+2\sqrt{7})(3-2\sqrt{7})$.

$$\boxed{-19}$$