Quiz 5

1. The joint density of X and Y is given by

$$f(x) = \begin{cases} x+y, & 0<x<1,\ 0<y<1 \\ 0, & \text{else} \end{cases}$$

(a) Are X and Y independent?

$$f_X(x) f_Y(y) = (x+1/2)(y+1/2)$$

$$= x^2 + \tfrac{1}{2}x + \tfrac{1}{2}y + \tfrac{1}{4}$$

$$\neq x+y \quad \text{No!}$$

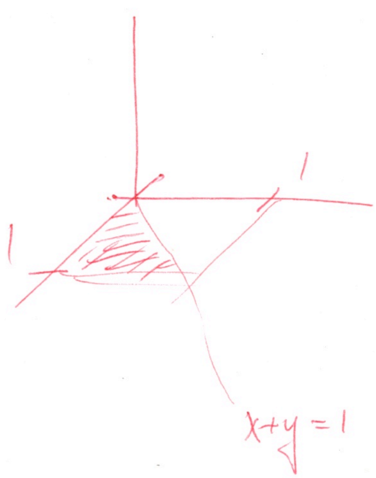
(b) Find the density function f_X of X.

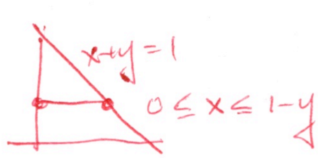
$$f_X(x) = \int_{\mathbb{R}} x+y \, dy = \int_0^1 x+y \, dy = x + \tfrac{1}{2}$$

$$f_Y(y) = \int_0^1 x+y \, dx = \tfrac{1}{2} + y$$

(c) Compute $P(X+Y<1)$. $= \iint_{x+y<1} x+y \, dA = \int_0^1 \int_0^{1-y} x+y \, dx \, dy$

$$= \int_0^1 \left[\tfrac{1}{2}x^2 + xy\right]_0^{1-y} dy$$





$$= \int_0^1 \tfrac{1}{2}(1-y)^2 + (1-y)y \, dy$$

$$= \int_0^1 \tfrac{1}{2} - \cancel{y} + \cancel{\tfrac{y^2}{2}} + \cancel{y} - \cancel{y^2} \, dy = \tfrac{1}{2} - \tfrac{1}{6}y^3\Big|_0^1$$

$$= \tfrac{2}{6} = \boxed{1/3}$$

$$X = \sum_{i=1}^{10} X_i \, , \quad X_i = \text{\# bet. } 1 \ \& \ 6$$

2. A fair die is rolled 10 times. Find the expected sum of the 10 rolls.

$$E[X] = \sum_{i=1}^{10} E[X_i]$$

$$= 10\,E[X_1]$$

$$= 10\left[\sum_{i=1}^{6} i\,p(i)\right]$$

$$= 10\left[1p(1) + 2p(2) + \cdots + 6p(6)\right]$$

$$= 10\left[\frac{1}{6} + 2\frac{1}{6} + \cdots + \frac{6}{6}\right]$$

$$\frac{21}{6} = 3.5$$

$$= \cancel{10}^{\,5} \cdot \frac{1}{6} \cdot \frac{\cancel{6}\cdot 7}{\cancel{2}} = \boxed{35}$$

$$\cancel{10 \cdot \frac{210}{36 \cdot 6} = \frac{105}{18} = \frac{35}{6}}$$