Quiz 3

1. Let X be the random variable counting the difference between the number of heads and the number of tails obtained when a coin is flipped n times. Find the range $R(X)$ of this random variable, and compute it's expectation $E[X]$ and variance $\mathrm{Var}(X)$ when $n = 5$.

$$R(X) = \{5-0, 4-1, 3-2, 2-3, 1-4, 0-5\}$$

$$= \{5, 3, 1, -1, -3, -5\}$$

$$p = 1 - p = \frac{1}{2}$$

$$\Rightarrow E[X] = \sum_{i \in R(X)} i p(i)$$

$$= 5P(X=5) + 3P(X=3) + 1P(X=1)$$

$$- 1P(X=-1) - 3P(X=-3) - 5P(X=-5)$$

$$= \cancel{5 \cdot \frac{1}{2^5}} + \cancel{3 \cdot \binom{5}{1}} \frac{1}{2^5} + \cancel{\binom{5}{2}} \cdot \frac{1}{2^5}$$

$$- \cancel{\binom{5}{3}} \frac{1}{2^5} - \cancel{3 \cdot \binom{5}{4}} \cdot \frac{1}{2^5} - \cancel{5 \cdot \frac{1}{2^5}}$$

$$= \boxed{0}$$

$$\mathrm{Var}(X) = E[X^2] - E[X]^2 = E[X^2]$$

$$= \left(25 \cdot \frac{1}{2^5} + 9 \cdot \binom{5}{1} \cdot \frac{1}{2^5} + 1 \cdot \binom{5}{2} \cdot \frac{1}{2^5}\right) \cdot 2$$

$$= \frac{(25 + 45 + 10) \cdot 2}{32} = \frac{160}{32} = \boxed{5}$$

2. Two coins are flipped. The first coin has $P(H) = 0.6$, and the second coin has $P(H) = 0.7$. Assuming the flips are independent and X is the random variable counting the number of heads, find the expectation $E[X]$.

$$E[X] = 0 \cdot p(0) + 1 \cdot p(1) + 2 \cdot p(2)$$

$$= 1 \cdot \left((0.6)(0.3) + (0.4)(0.7)\right) + 2 \cdot \left((0.6)(0.7)\right)$$

$$= \frac{18 + 28 + 84}{100}$$

$$= \frac{130}{100}$$

$$= \boxed{1.3}$$