Quiz 1

1. A committee of 7, consisting of 2 Republicans, 2 Democrats, and 3 Independents, is to be chosen from a group of 5 Republicans, 6 Democrats, and 4 Independents. How many committees are possible?

$$\binom{5}{2}\binom{6}{2}\binom{4}{3} = \frac{5\cdot 4}{2}\cdot\frac{6\cdot 5}{3}\cdot 4$$

$$= \boxed{600}$$

2. How may vectors $(x_1, \ldots, x_k)$ are there satisfying:

   - Each x_i is an integer between 1 and n.
   - $x_1 < x_2 < \cdots < x_k$.

There are $\boxed{\binom{n}{k}}$ ways to pick k of n slots among slots

$\binom{5}{2} = \frac{5 \cdot 4}{2} = 10$

3. Expand $(2x^3 + y)^5$ using the binomial theorem.

$= (2x^3)^5 + 5(2x^3)^4 y + 10(2x^3)^3 y^2 + 10(2x^3)^2 y^3$

$+ 5(2x^3) y^4 + y^5$

$= 32x^{15} + 80x^{12} y + 80x^9 y^2 + 40x^6 y^3 + 10x^3 y^4 + y^5$

4. Expand $(x + 2y + 3z)^4$ using the multinomial theorem.

$\binom{6}{2} = \frac{6 \cdot 5}{2} = 15$ ✓

$\binom{4}{4,0,0} = \binom{4}{0,4,0} = \binom{4}{0,0,4} = 1$

$\binom{4}{3,1,0} = \binom{4}{3,0,1} = \binom{4}{1,3,0} = \binom{4}{0,3,1} = \binom{4}{1,0,3} = \binom{4}{0,1,3} = 4$

$\binom{4}{2,1,1} = \binom{4}{1,2,1} = \binom{4}{1,1,2} = 6$

$\binom{4}{2,2,0} = \binom{4}{2,0,2} = \binom{4}{0,2,2} = 6$

$\Rightarrow (x+2y+3z)^4 = x^4 + (2y)^4 + (3z)^4 + 4x^3(2y) + 4x^3(3z)$

$+ 4x(2y)^3 + 4(2y)^3(3z) + 4 \cdot x \cdot (3z)^3 + 4(2y)(3z)^3$

$+ 6x^2(2y)(3z) + 6x(2y)^2(3z) + 6x(2y)(3z)^2$

$+ 6x^2(2y)^2 + 6x^2(3z)^2 + 6(2y)^2(3z)^2$