

Quiz 9

MATH 2300-001

October 21, 2008

1. For what values of x does the series $\sum_{k=0}^{\infty} 5e^{-kx^2}$ converge, and what does the series converge to?

$$\sum_{k=0}^{\infty} 5e^{-kx^2} = \sum_{k=0}^{\infty} 5 \left(e^{-x^2} \right)^k = \frac{5}{1 - e^{-x^2}}$$

This convergence holds for

$$|e^{-x^2}| < 1$$

$$e^{-x^2} < 1$$

$$-x^2 < 0$$

$$x^2 > 0$$

$$|x| > 0$$

$$x \neq 0.$$

2. Suppose $\sum_{k=1}^n a_k = 5 - \frac{9}{n}$ for all $n \geq 1$.

(a) Find a_{100} .

(b) If the series $\sum_{k=1}^{\infty} a_k$ converges, find its sum. Otherwise, explain why it diverges.

$$(a) \quad a_{100} = \sum_{k=1}^{100} a_k - \sum_{k=1}^{99} a_k = \left(5 - \frac{9}{100} \right) - \left(5 - \frac{9}{99} \right) = -\frac{9}{100} + \frac{1}{11} = \frac{1}{1100}.$$

$$(b) \quad \sum_{k=1}^{\infty} a_k = \lim_{n \rightarrow \infty} \sum_{k=1}^n a_k = \lim_{n \rightarrow \infty} \left(5 - \frac{9}{n} \right) = 5.$$