Quiz 8

MATH 2300-001
October 14, 2008

1. Show that $\displaystyle\int_{-\infty}^{\infty} \frac{1}{a^2+x^2}\,dx = \frac{\pi}{a}$ for all $a > 0$.

$$\int_0^b \frac{1}{a^2+x^2}\,dx = \frac{1}{a^2}\int_0^b \frac{1}{1+\left(\frac{x}{a}\right)^2}\,dx = \frac{1}{a}\left[\tan^{-1}\left(\frac{x}{a}\right)\right]_0^b = \frac{1}{a}\tan^{-1}\left(\frac{b}{a}\right).$$

$$\int_0^\infty \frac{1}{a^2+x^2}\,dx = \lim_{b\to\infty}\int_0^b \frac{1}{a^2+x^2}\,dx = \lim_{b\to\infty}\frac{1}{a}\tan^{-1}\left(\frac{b}{a}\right) = \lim_{c\to\infty}\frac{1}{a}\tan^{-1} c = \frac{\pi}{2a}.$$

$$\int_{-\infty}^0 \frac{1}{a^2+x^2}\,dx = \lim_{b\to\infty}\int_{-b}^0 \frac{1}{a^2+x^2}\,dx = -\lim_{b\to\infty}\int_b^0 \frac{1}{a^2+x^2}\,dx = \lim_{b\to\infty}\int_0^b \frac{1}{a^2+x^2}\,dx$$
$$= \frac{\pi}{2a}.$$

$$\int_{-\infty}^\infty \frac{1}{a^2+x^2}\,dx = \int_{-\infty}^0 \frac{1}{a^2+x^2}\,dx + \int_0^\infty \frac{1}{a^2+x^2}\,dx = \frac{\pi}{2a} + \frac{\pi}{2a} = \frac{\pi}{a}.$$

OR

$$\int_{-\infty}^\infty \frac{1}{a^2+x^2}\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{a\sec^2\theta}{a^2+a^2\tan^2\theta}\,d\theta = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{a}\,d\theta = \frac{\pi}{a}.$$

2. Give an example of each of the following:

   (a) A convergent sequence that is not monotone.

   (b) A monotone sequence that is not convergent.

   (You do not need to verify your examples satisfy the conditions.)

   (a) $\left\{\dfrac{(-1)^n}{n}\right\}_{n=1}^{\infty}$

   (b) $\{n\}_{n=1}^{\infty}$