Quiz 7

MATH 2300-001
October 7, 2008

1. For the sequence $\{a_n\}_{n=1}^{\infty} = \{\int_1^n \frac{1}{x^2}\,dx\}_{n=1}^{\infty}$:

   (a) List the first five terms of the sequence, in simplified form.

   (b) State the limit of the sequence, if it exists.

$$a_n = \int_1^n \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^n = 1 - \frac{1}{n} = \frac{n-1}{n}$$

$$a_1 = \frac{0}{1} = 0, \quad a_2 = \frac{1}{2}, \quad a_3 = \frac{2}{3}, \quad a_4 = \frac{3}{4}, \quad a_5 = \frac{4}{5}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1 - \frac{1}{n}\right) = 1 - 0 = 1.$$

2. State a differential equation that has solutions $y_1 = \pi e^{-5x}$ and $y_2 = 26xe^{-5x}$.

   Note the the solutions are of the form c_1e^{ax}, c_2xe^{ax}, which comes from the auxiliary equation $(k-a)^2 = 0 \Rightarrow k^2 - 2ak + a^2 = 0$. This would give the differential equation $y'' - 2ay' + a^2y = 0$.

   Since we have $a = -5$, the differential equation is $y'' + 10y' + 25y = 0$.