Quiz 6

MATH 2300-001
September 30, 2008

1. For what real numbers p does the integral $\int_0^\infty \frac{1}{x^{2p}}\,dx$ converge? Diverge? Explain. *(You may use any results stated in class).*

   For $2p \geq 1\,(p \geq \frac{1}{2})$, $\int_0^1 \frac{1}{x^{2p}}\,dx$ diverges.

   For $2p \leq 1\,(p \leq \frac{1}{2})$, $\int_1^\infty \frac{1}{x^{2p}}\,dx$ diverges.

   So, $\int_0^\infty \frac{1}{x^{2p}}\,dx = \int_0^1 \frac{1}{x^{2p}}\,dx + \int_1^\infty \frac{1}{x^{2p}}\,dx$ must diverge for all $p \in \mathbb{R}$.

2. Verify that $y(x) = e^{\frac{x^2}{2}}$ is a solution to the ODE $y'' - xy' - y = 0$.

$$
\begin{aligned}
y(x) &= e^{\frac{x^2}{2}} \\
y'(x) &= xe^{\frac{x^2}{2}} \\
y''(x) &= (x^2+1)e^{\frac{x^2}{2}}
\end{aligned}
$$

$$y'' - xy' - y = e^{\frac{x^2}{2}}(x^2 + 1 - x^2 - 1) = 0.$$