

Quiz 5

MATH 2300-001

September 23, 2008

1. $\int_{\sqrt{2}}^2 \frac{dx}{\sqrt{x^2 - 2}} =$

$$x = \sqrt{2} \sec \theta$$

$$dx = \sqrt{2} \sec \theta \tan \theta d\theta$$

$$\begin{aligned} \int_{\sqrt{2}}^2 \frac{dx}{\sqrt{x^2 - 2}} &= \int_0^{\frac{\pi}{4}} \frac{\sqrt{2} \sec \theta \tan \theta}{\sqrt{2 - 2 \sec^2 \theta}} d\theta \\ &= \int_0^{\frac{\pi}{4}} \frac{\sqrt{2} \sec \theta \tan \theta}{\sqrt{2} \tan \theta} d\theta \\ &= \int_0^{\frac{\pi}{4}} \sec \theta d\theta \\ &= [\ln |\sec \theta + \tan \theta|]_0^{\frac{\pi}{4}} \\ &= \ln(\sqrt{2} + 1) - \ln(1 + 0) \\ &= \ln(\sqrt{2} + 1). \end{aligned}$$

2. $\int \frac{x^2 + x - 1}{(x - 2)(x^2 + 1)} dx =$

$$\begin{aligned} \frac{x^2 + x - 1}{(x - 2)(x^2 + 1)} &= \frac{A}{x - 2} + \frac{Bx + C}{x^2 + 1} \\ x^2 + x - 1 &= A(x^2 + 1) + (Bx + C)(x - 2) \\ &= (A + B)x^2 + (C - 2B)x + A - 2C \end{aligned}$$

$$\left. \begin{array}{l} 1 = A + B \\ 1 = C - 2B \end{array} \right\} \Rightarrow 3 = 2A + C$$
$$\left. \begin{array}{l} -1 = A - 2C \\ 3 = 2A + C \end{array} \right\} \Rightarrow 5 = 5A \Rightarrow A = 1 \Rightarrow B = 0, C = 1$$

$$\begin{aligned} \int \frac{x^2 + x - 1}{(x - 2)(x^2 + 1)} dx &= \int \frac{1}{x - 2} + \frac{1}{1 + x^2} dx \\ &= \ln |x - 2| + \tan^{-1} x + C. \end{aligned}$$