

Quiz 4

MATH 2300-001

September 15, 2008

1. Find the arc length of $f(x) = \int_1^{x^2} \frac{\sqrt{t-1}}{2\sqrt{t}} dt$ over the interval $[2,6]$.

$$\begin{aligned} L &= \int_2^6 \sqrt{1 + (f'(x))^2} dx \\ &= \int_2^6 \sqrt{1 + \left(\frac{\sqrt{x^2-1}}{2x} 2x \right)^2} dx \\ &= \int_2^6 \sqrt{1 + x^2 - 1} dx \\ &= \int_2^6 x dx \\ &= \left[\frac{1}{2} x^2 \right]_2^6 \\ &= \frac{1}{2} (36 - 4) \\ &= 16. \end{aligned}$$

2. $\int \sinh^3 x \cosh^3 x dx =$

$$\begin{aligned} &\int \sinh^3 x (1 + \sinh^2 x) \cosh x dx \\ &= \int u^3 (1 + u^2) du \\ &= \int u^3 + u^5 du \\ &= \frac{1}{4} u^4 + \frac{1}{6} u^6 + C \\ &= \frac{1}{4} \sinh^4 x + \frac{1}{6} \sinh^6 x + C \\ &\left(= -\frac{1}{4} \cosh^4 x + \frac{1}{6} \cosh^6 x + C \right). \end{aligned}$$