Quiz 3

MATH 2300-001

September 9, 2008

1. Show that $\sinh^2 x + \cosh^2 x = \cosh(2x)$.

$$
\begin{aligned}
\cosh^2 x + \sinh^2 x &= \left(\frac{e^x + e^{-x}}{2}\right)^2 + \left(\frac{e^x - e^{-x}}{2}\right)^2 \\
&= \frac{e^{2x} + 2 + e^{-2x}}{4} + \frac{e^{2x} - 2 + e^{-2x}}{4} \\
&= \frac{2e^{2x} + 2e^{-2x}}{4} \\
&= \frac{e^{2x} + e^{-2x}}{2} \\
&= \cosh(2x).
\end{aligned}
$$

2. $\displaystyle \int \sin^{-1} x \, dx =$

$$
\begin{aligned}
u = \sin^{-1} x \qquad &\Rightarrow \qquad du = \frac{1}{\sqrt{1 - x^2}} \, dx \\
dv = dx \qquad &\Rightarrow \qquad v = x
\end{aligned}
$$

$$
\begin{aligned}
\int \sin^{-1} x \, dx &= x \sin^{-1} x - \int \frac{x}{\sqrt{1 - x^2}} \, dx \\
&= x \sin^{-1} x + \int \frac{1}{2} \cdot \frac{1}{\sqrt{u}} \, du \\
&= x \sin^{-1} x + \sqrt{1 - x^2} + C.
\end{aligned}
$$